Characterization and Modeling of Microstructures Using Volume Images

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Methods from integral geometry and mathematical morphology yield efficient algorithms for the geometric characterization of microstructures using 3d images. In particular, the intrinsic volumes and their densities are used to describe objects like pores or cells and components, respectively.

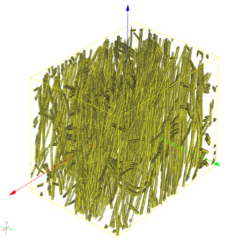

Figure 1. Synchrotron CT-reconstruction of a glass fiber-reinforced polymer (GRP), 3.5µm voxel size. Sample: *IVW Kaiserslautern*, Imaging: *ESRF Grenoble.*

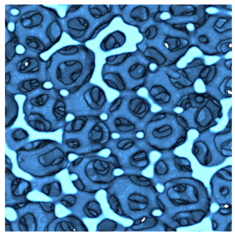

Figure 2. CT-reconstruction of an open aluminum foam, 64.57µm voxel size.

Intrinsic Volumes as Geometric Characteristics

The Minkowski functionals (i.e. volume V, surface area S, integral of mean curvature M and Euler number χ) and their densities V_V, S_V, M_V and χ_V serve as a basic set of characteristics for microstructures, e.g. the solid components of open foams or the fiber systems in fiber-reinforced materials. Their estimation requires only a simple binarization of the foam structure and can be computed efficiently from µCT-images using discrete versions of the Crofton and Euler-Poincaré formulae [1].

Fibrous Materials

The geometry of the fiber system influences a fiber-reinforced material's mechanical properties. Besides the Minkowski-functionals and some derived quantities (Tab. 1), an important factor is the **fiber orientation distribution**, which is contained in the generalized projections needed to compute the integral of mean curvature M_V. The orientation distribution ϑ in the typical fiber point is related to these generalized projections by

$$\wp_M^{\omega}(X) = \int_{S^2} |\langle \omega, u \rangle| \, \vartheta(du).$$

Thus, ϑ can be computed by inverting this cosine transform.

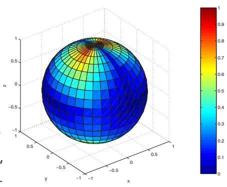

Figure 3. The orientation distribtion ϑ of the GRP in Fig. 1, computed in 13 discrete directions (x,y,z, 6 face diagonals, 4 space diagonals).

Table 1. Estimates of geometric characteristics of the fiber-reinforced polymer shown in Figure 1.

V_V		11.30	%
S_V		14.20	mm^{-1}
M_V		485.53	mm^{-2}
porosity	$1-V_V$	88.70	%
specific fiber length	$M_V/(\pi(1-V_V))$	154.55	mm^{-2}

Open Foams

Similar to fiber systems, the Minkowski-functionals also characterize the geometry of the strut systems of open foams (Tab. 2). Especially for subsequent modelling steps, it is important to have access to geometric characteristics of the struts, not only to those of the tessellation [2].

Table 2. Estimates of geometric characteristics of the aluminum foam shown in Figure 2.

V_V		10.56	%
S_V		0.533	mm^{-1}
M_V		0.543	mm^{-2}
χ_V		-0.079	mm^{-3}
porosity	$1-V_V$	89.44	%
strut length density	$M_V/(\pi(1-V_V))$	0.193	mm^{-2}
mean strut diameter	$S_V/(\pi L_V)$	0.878	mm
mean strut perimeter	S_V/L_V	2.758	mm

Stochastic Models for Virtual Material Design

Interpreting the relevant component of a material as a stationary random closed set (RACS) leads to manifold stochastic material models. E.g., particle processes for fiber systems in fiber-reinforced polymers and the edge system of random tessellations for open foams. Using estimated characteristics as fitting parameters, stochastic models are fitted to the observed structures [3,4,5]. Simulations of physical properties in both the original samples and models with altered microstructure allow to study relations between the geometry of foams or fibers and their macroscopic behavior. This allows for a »virtual« design and optimization of materials for particular applications.

Fibrous Materials

We fit models of fiber systems (random sequential adsorption of non-overlapping cylinders) to the volume density V_V and orientation distribution ϑ measured from tomographic images [4].

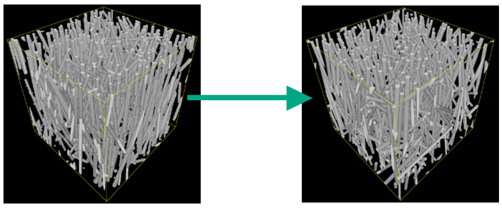

Figure 4. RSA-model of non-overlapping cylinders fit to a glass fiber-reinforced polymer.

Open Foams

Random Laguerre tessellations, a weighted form of the Voronoi tessallations, have proven to be suitable models for the edge system of open foams [3,5]. They are very flexible and allow to generate a large variety of cell structures. In particular, tessellations generated from dense packings of spheres model real foams very well. If required, the shape of the struts can be modified using locally adaptive morphology [5].

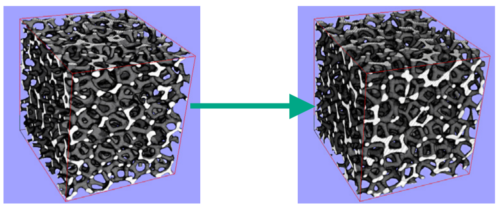

Figure 5. Dilated edge system of a random Laguerre tessellation fit to the open aluminum foam from Fig. 2.

References

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[2] K. Schladitz, C. Redenbach, T. Sych, M. Godehardt, Report 148, Fraunhofer ITWM, Kaiserslautern, 2008.
[3] C. Lautensack, J. Appl. Stat. 35(9), 2008
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