A multiscale modelling of fluid transport within bones: consequences of electrokinetics in the mechanotransduction of bone remodelling signals

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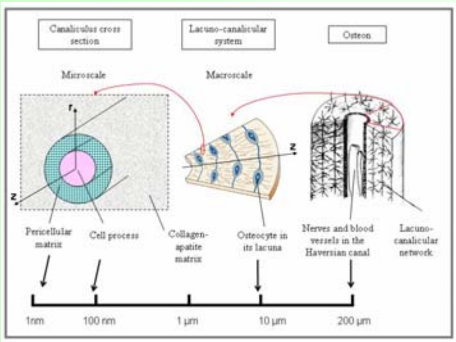
A two-scale modelling is carried out to describe the hydro-electro-chemical behaviour of a saturated porous medium. Thus, the electro-viscous approach of Moyne and Murad is changed to take into account submicroscopic friction effects generated by the nanometric pericellular fibers. As a result, the fluid movement at the pore's scale is described by combining a Brinkman law with chemical transport and electrostatics equations. Using a suitable change of variables, the problem is upscaled thanks to a periodic homogenization procedure to derive the macroscopic description. For instance, the consequences of electrokinetics in the mechanotransduction of bone remodelling signals can be discussed thanks to the obtained coupled Darcy law.

Multiscale structure of cortical bone



Modelling at the canalicular scale

Electrostatics (1)

$$\nabla\cdot\nabla\phi = -\frac{F}{\varepsilon\varepsilon_0}(n^+ - n^-)$$ at the wall : $$\nabla\phi\cdot\mathbf{n} = \frac{\sigma}{\varepsilon\varepsilon_0}$$

Ionic transport (2)

$$\frac{\partial n^\pm}{\partial t} + \nabla\cdot(n^\pm\mathbf{v}) = \nabla\cdot[D_\pm\exp(\mp\bar\phi)\nabla(n^\pm\exp(\pm\bar\phi))]$$

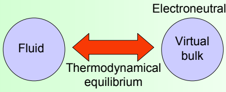
at the wall : $$-D_\pm(\nabla n^\pm \pm n^\pm\nabla\bar\phi)\cdot\mathbf{n} = 0$$

Fluid movement (3)

$$-\nabla p + \mu_f\nabla\cdot\nabla\mathbf{v} - \frac{\mu_f}{k_f}\mathbf{v} = F(n^+ - n^-)\nabla\phi$$ at the wall : no-slip

Brinkman term including k_f, a permeability parameter representing the submicroscopical effects of the pericellular network.

Virtual bulk variables



$\phi \; n^+ \; n^- \; p$

1 electrical potential
2 concentrations
1 pressure

1 bulk electrical potential ψ_b
1 double-layer electrical potential φ
1 bulk concentration n_b
1 bulk pressure p_b

$$\phi = \psi_b + \varphi,\quad n^\pm = n_b\exp(\mp\bar\varphi),\quad p_b = p - \pi = p - 2RTn_b(\cosh\bar\varphi - 1)$$

Rewriting of the micromodel

(1) $$\Delta(\bar\psi_b + \bar\varphi) = \frac{1}{L_D^2}\sinh\bar\varphi$$ where $\bar\blacksquare = F\blacksquare/RT$, $L_D = \sqrt{\frac{\varepsilon\varepsilon_0 RT}{2F^2 n_b}}$, Debye length.

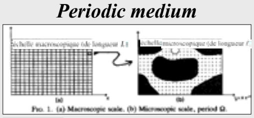
(2) $$\frac{\partial(n_b\exp(\mp\bar\varphi))}{\partial t} + \nabla\cdot(n_b\exp(\mp\bar\varphi)\mathbf{v}) = \nabla\cdot(D_\pm(\exp(\mp\bar\varphi)(\nabla n_b \pm n_b\nabla\bar\psi_b)))$$

(3) $$\mu_f\nabla\cdot\nabla\mathbf{v} - \frac{\mu_f}{k_f}\mathbf{v} - \nabla p_b - 2RT(\cosh\bar\varphi - 1)\nabla n_b + 2RTn_b\sinh\bar\varphi\nabla\bar\psi_b = 0$$

Homogenization of the fluid movement equation

Foreword : on this poster, only the upscaling procedure of the fluid movement equation is presented. The Poisson-Boltzmann and Nernst-Planck equations are treated elsewhere as proposed by Moyne and Murad (Moyne and Murad 2002, Lemaire et al. 2008).

Periodic medium



$\eta = \ell/L$ We assume cylindrical pores.

Space derivating operator

$L_r \equiv L \longrightarrow \nabla = \frac{1}{L}\nabla'$

radially $x_r = \ell$

longitudinally $X_r = L$

$\longrightarrow \nabla' = \eta^{-1}\nabla'_x + \nabla'_X$

Reference values

$F\phi_r/RT \equiv O(\eta^0)$

p_r $\quad n_r \equiv \varepsilon\varepsilon_0\phi_r/F\ell^2$ $\quad v_r$

$k_f \equiv O(\ell^2)$

Scaling laws

Reynolds $Re = p_r L/\mu_f v_r \equiv O(\eta^{-2})$

Derjaguin $De = 2RTn_r/p_r \equiv O(1)$

$\mathbf{v} = u(r)\mathbf{e}_z$

Asymptotic expansion and closure problems.

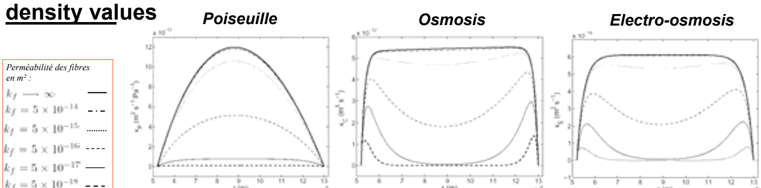
3 problems to solve

$$\mu_f\left(\frac{d^2u_\alpha}{dr^2} + \frac{1}{r}\frac{du_\alpha}{dr}\right) - \frac{\mu_f}{k_f}u_\alpha = G_\alpha F_\alpha(r)$$

where $\alpha = P, C$ **&** E **represent respectively the Poiseuille, chemical and electro-osmotic effects.** G_α **&** F_α **are the following functions:**

$$G_P = \frac{dp_b}{dz}, F_P(r) = 1, G_C = \frac{dn_b}{dz}, F_C(r) = 2RT(\cosh\bar\varphi - 1), G_E = \frac{d\bar\psi_b}{dz}, F_E(r) = -2RTn_b\sinh\bar\varphi$$

Results : local permeabilty parameters for various pericellular matrix density values



- Parabolic profiles
- Slowdown due to the pericellular matrix

- Flat profiles with over-velocities in the double-layers

Conclusions

- *Coupled model of bone interstitial fluid flow.*
- *Shear stress is a key-element of the mechanotransduction of bone remodelling (Klein-Nulend et al.)*
- *With our coupled model, we can evaluate the coupled shear effects*

Osmosis	Poiseuille	Electro-osmosis
$S_C \sim 4$ Pa	$S_P \sim 1$ Pa	$S_E \sim 1$ Pa