Mathematical modelization for computing the permeability in human cortical bone

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OBJECTIVES

Cortical bone is more and more considered as a porous medium and this point of view involves the determination of the porosity and the permeability. Porosity does not present a major problem, at least for the order of magnitude. There is a difficulty for the permeability: according to literature, experimental values vary between 10^{-13} and 10^{-15} m² [1,2]. A study based on a concept of multi scale analysis corresponding to the scales already introduced in the SiNuPrOs modelization [3] is performed to compute the permeability tensor in various configurations.

Experimental measurements

K_{11} = 1.1 x 10^{-13} m ²

[1] Malachanne E. et al, "Experimental and numerical identification of cortical bone permeability", J. of Biomechanics, Vol. 41, Issue 3, 721-725, 2008

K_{11} = 5 x 10^{-15} m ²

[2] Li G. et al. "Permeability of cortical bone of canine tibiae", Microvascular Research 34, 302-310, 1987

METHODS

Structural analysis of cortical bone at the macroscopic scale highlights three types of networks containing fluid and having different sizes. The first one is composed by the Havers and Volkmann channels, the second one is composed by the canaliculae while the third one is constituted by the interstices between the Hap crystals when the mineralization is not yet sufficiently advanced. The osteocytar volume is not taken into account because its only function is to lodge a cell; it isn't a pathway for the fluid.

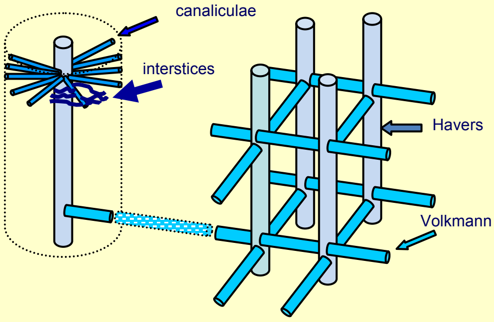
The fluid flow is modeled by the Stokes equations. The homogenization theory [4] allows showing that the homogenized behavior's law of such a fluid is a Darcy's law. Moreover, it is possible to prove that the macroscopic permeability can be obtained from the solution of the cell problem:

$$\begin{cases} \Delta w_j(y) = \nabla_y \pi_j(y) - e_j & \text{in } Y_f \\ \nabla \cdot w_j(y) = 0 & \text{in } Y_f \\ w_j = 0 & \text{on } \Gamma \end{cases}$$

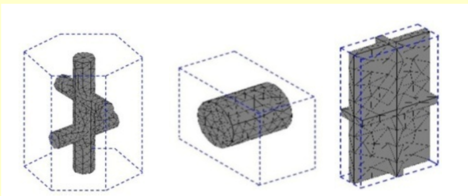
where Y_f is the fluid part of the basic cell Y, Γ is the interface between solid and fluid, w_j are Y-periodic velocity vector fields, π_j are Y-periodic pressures fields and $(e_j)_{1\le j\le 3}$ is the canonical basis of R^3. The permeability tensor is given by

$$K_{ij} = \frac{1}{|Y|}\int_{Y_f} w_{ji}(y)\,\mathrm{d}y \qquad i, j = \overline{1,3}$$

A numerical scheme was implemented in Matlab to get the K_{ij} values. Since many architectural configurations are possible, a reference configuration is associated to the SiNuPrOs modelization. For each scale different basic cells have been used.



Fluid architectural organization



For the macroscopic permeability | For the osteonal permeability | For the lamellar permeability

Various meshes used to determine the permeability

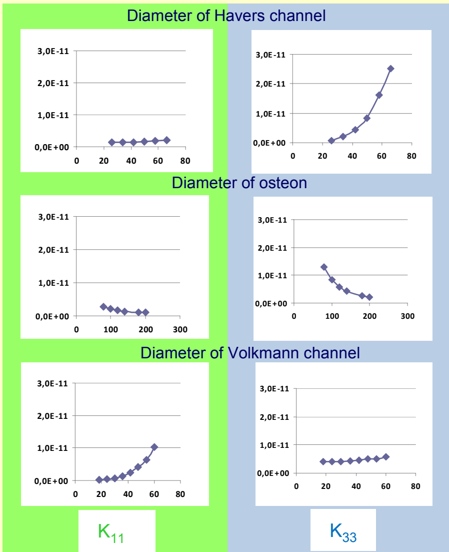
RESULTS

A first study is devoted to the permeabilities at each scale and one notes different order of magnitude (in m²): 10^{-12} for the macroscopic scale, 10^{-15} for the osteonal one and 10^{-20} for the lamellar one.

It is possible to determine configurations for which the theoretical value of the permeability approaches the experimental value [1].

diameter of Havers	diameter of Volkmann	dist. between two Volkmann	diameter of osteon	dist. between two osteons	K_{11}(m²)
50 µm	24 µm	600 µm	170 µm	1 µm	1.2 x 10^{-13}
54µm	24.172 µm	550 µm	190 µm	10 µm	1.1 x 10^{-13}

Concerning the effects, at the macroscopic level, of variations of the parameters related to the fluid modelization an analysis shows that the most important parameters for the variations of the K_{33} coefficient are successively the diameter of the haversian channel and the diameter of the osteon. For the K_{11} (or K_{22}) coefficient, the important parameter is the diameter of the Volkmann channel .



Effect of the main architectural parameters on the permeability

CONCLUSIONS

This modelization allows computing all the permeability tensor coefficients and investigations for comparative analyses through various parametric studies. The permeability coefficients can be in good correlation with experimental data for particular configurations. The orders of magnitude depend strongly on the considered scale.

REFERENCES

[3] Predoi-Racila M., Crolet J.M., Human cortical bone: the SINUPROS model. Part I - Description and macroscopic results, CMBBE, Vol. 11, 169 – 187, 2008

[4] Sanchez-Palencia E., Non homogeneous media and vibration theory, 129, Lecture notes in Physics, Springer Verlag Berlin, 1980