Morphological measurements

for Random Structures

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Introduction

Microstructure of materials: **heterogeneity**

Qualitative and **quantitative** Analysis of the morphology (**images analysis)**

$\Downarrow$

Construction of **probabilistic models** of structures for porous or multicomponent media

$\Downarrow$

Understanding and prediction of the behaviour of materials in service

Types of random structures

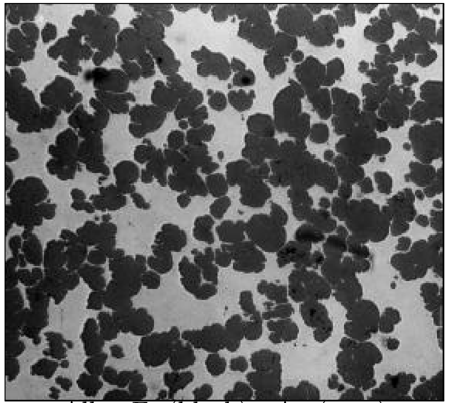
Principal types of regionalized data that used in materials:

- i) Dispersions of small particles in a matrix (defects like non metallic inclusions in steel, damage initiation,...), → **stochastic point processes.**
- ii) Granular structures (polycrystals), assimilated to **random tessellations of space** (each class corresponding to a grain).
- iii) Two phase (porous media) or multiphase structures → **random sets** (binary), or **multicomponent random sets**

Types of random structures

- iv) Rough surfaces (steel plate, fracture surface,...), chemical concentration mappings (X ray images), grey level images (video images, secondary electron images in a scanning electron microscope, local water content in wood...) → **random functions.**
- v) Multivariate data (multi species chemical mappings, components of a vector or of a tensor in every point x of space), → **multivariate random function models.**
- vii) Data on a network connecting vertices (roads, porous medium, cracks,...) → **random graphs.**

Example of two phase random structures



Alloy Fe (black) - Ag (grey)

Characterization of a random structure

Characterization of a random structure

Main Criteria

- Basic measures (volume fraction V_V, surface area S_V, integral of mean curvature M_V,...) $\rightarrow$ stereology
- Size distribution (2D-3D)
- Distribution in space:
  - Clustering
  - Scales
  - Anisotropy
- Connectivity

Principle of morphological Measurements

Two steps (J. Serra):

- **morphological transformation** Φ applied to the structure (use of image analysis)
- **measurement** performed on the transformed object

Choice of measures and transformations according to morphological criteria

Principle of morphological Measurements

Experimental access → **constraints** for allowed measurements:

- invariance by translation
- continuity (with respect to the mesh and to the sampling grid)
- local knowledge (study of the structure from bounded measure fields)
- additivity (averages)
- stereological properties (2D-3D)

Measurements and transformations respecting these constraints.

Basic measurements and Minkowski functionals

In integral geometry, it is shown that in R^n, $n+1$ measures satisfy the constraints: the de Minkowski functionals, noted W_i, with W_i homogeneous and of degree $n-i$: $W_i(\lambda A) = \lambda^{n-i} W_i(A)$.

In R

$$\begin{aligned} W_0(A) &= l(A) \text{ (length of } A) \\ W_1(A) &= 2 \end{aligned} \tag{1}$$

Basic measurements and Minkowski functionals

In R^2

$$\begin{aligned} W_0(A) &= A(A) \text{ (area of the set } A) \\ 2W_1(A) &= L(A) \text{ (perimeter of } A) \\ 2W_2(A) &= 2\pi N(A) \text{ (connectivity number in } R^2) \end{aligned} \tag{2}$$

$N(A)$: difference between the number of connected components of A and the number of holes it contains.

Basic measurements and Minkowski functionals

In R^3

$$\begin{aligned} W_0(A) &= V(A) \text{ (volume of the set } A) \\ 3W_1(A) &= S(A) \text{ (surface area of } A) \\ 3W_2(A) &= M(A) = \frac{1}{2}\int_{\partial A}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) dS \\ &\text{(integral of the mean curvature)} \\ 3W_3(A) &= 4\pi(N-G) = N(A) = \int_{\partial A}\left(\frac{1}{R_1 R_2}\right) dS \\ &\text{(integral of the total curvature,} \\ &\text{or connectivity number in } R^3) \end{aligned} \tag{3}$$

Basic measurements and Minkowski functionals

R_1 and R_2: principal radii of curvature in any point of the boundary ∂A of A.

$(N - G)$: difference between the number of connected components of A, N and its genus G.

Genus of A : maximal number of closed curves to be drawn on its boundary ∂A without disconnecting it into two parts: 0 for a sphere and 1 for a torus.

Connectivity numbers

Connectivity numbers in R^2 and in R^3: **topological characteristics** describing the connectivity of an object

in R^2, $N(A)$ is obtained by the difference between two **convexity numbers** $C(A)$ and $C(A^c)$:

$$N(A) = C(A) - C(A^c) = \frac{1}{2\pi}\left(\int_{R>0} d\alpha - \int_{R<0} d\alpha\right) \quad (4)$$

R : radius of curvature in every point of ∂A and $d\alpha = \frac{ds}{R}$ rotation of the normal along the boundary, according to the arc ds

Morphological criteria

Basic Specific Measurements in R^3

For a random set A, the functionals W_i are random variables.

For a stationary random set (porous medium, mineralogical texture),with average properties invariant by translation, **specific values,** given per unit volume

- volume fraction V_V
- specific surface area S_V
- integral of the mean curvature M_V or of the total curvature $N_V - G_V$

estimation from slices (1D or 2D)

Specific Measurements and Stereology

Stereological	relationships	(Crofton)	for Slices	
R^3	V_V	S_V	M_V	$N_V - G_V$
	$\uparrow$	$\uparrow$	$\uparrow$	
	$V_V = \overline{A}_A$	$S_V = \frac{4}{\pi}\overline{L}_A$	$M_V = 2\pi\overline{N}_A$	
	$\uparrow$	$\uparrow$	$\uparrow$	
R^2	A_A	L_A	N_A	
	$\uparrow$	$\uparrow$		
	$A_A = \overline{L}_L$	$L_A = \pi\overline{N}_L$		
	$\uparrow$	$\uparrow$		
R	L_L	N_L		

Counting

Measurements estimated from images sampled on a **grid of points**, usually regular (square, or hexagonal in the plane), related to a graph.

- To every node x of the grid is given the value 1 if $x \in A$, and 0 if $x \in A^c$ (complementary set of A).
- **Number of summits** v of the grid in $A \rightarrow$ **area** in R^2 or **volume** in R^3
- **Number of intercepts** $N(01)$ in a given direction $\rightarrow$ estimation of the number of chords generated by A in this direction

Counting

- **Connectivity Numbers** estimated by counting from the Euler relations
- In the plane, N, v, the number of faces (f) and the number of edges (e) are related by:

$$N = v - e + f \tag{6}$$

On a hexagonal or a square grid, (6) is deduced from counting the following configurations (where * means "not tested"):

$$N(A) = N\binom{00}{1} - N\binom{0}{11} \text{ on the hexagonal grid}$$
$$(7)$$
$$N(A) = N\binom{10}{00} - N\binom{*1}{10} \text{ on the square grid}$$

Counting

In R^3, the Euler relation is expressed as a function of previous properties and of the number of blocks (b):

$$N - G = v - e + f - b \tag{8}$$

For an isotropic structure sampled on a cubic grid, specific connectivity number estimated from relation (8):

$$N_V - G_V = -\lim_{a \to 0} \left(\frac{C(0) - 3C(a) + 3P\binom{11}{11} - P(C)}{a^3} \right) \tag{9}$$

where C is the cube with side a.

Basic Operations

of Mathematical Morphology

Qualitative description of the morphology of objects: spherical or elongated pore, polyhedral grain...

- **Structuring element**: choice of an object K (compact set, like a point, sphere, segment) implanted in every point x of the euclidean space R^n
- **Answer to a question** on the mutual location of K_x and of the studied set A, for every point x
- For a **binary answer**, **indicator function** of $\Phi(A)$ obtained by transformation of the set A ($k(x) = 1$ for $x \in \Phi(A)$, else $k(x) = 0$)

Basic Operations

of Mathematical Morphology

Dilation and erosion of A by K:

$$\begin{array}{c} \text{i) } K \text{ hits } A? \ (K \cap A \neq \emptyset)? \\ \text{ii) } K \text{ included in } A \ (K \subset A)? \end{array} \quad (10)$$

$$A \oplus \check{K} = \{x, K_x \cap A \neq \emptyset\} = \cup_{y \in K} A_{-y} = \cup_{x \in A, y \in K} \{x - y\} \quad (11)$$

$$A \ominus \check{K} = \{x, K_x \subset A\} = \cap_{y \in K} A_{-y} = (A^c \oplus \check{K})^c \quad (12)$$

$K_x = \{x + y, y \in K\}$: translated of K to point x; $\check{K}$ obtained by transposition of K: $\check{K} = \{-x, x \in K\}$

Basic Operations

of Mathematical Morphology

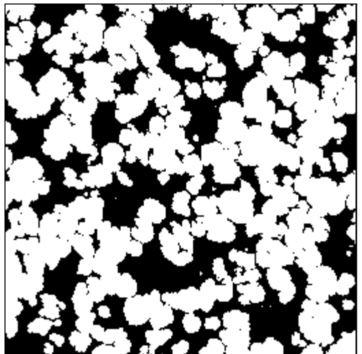
Symbols $\oplus$ and $\ominus$: Minkowski **addition** and **substraction** :

$$A \oplus K = \cup_{x \in A, y \in K} \{x + y\} = \cup_{y \in K} A_y = \cup_{x \in A} K_x \quad (13)$$

$$A \ominus K = \cap_{y \in K} A_y = (A^c \oplus K)^c \quad (14)$$

Basic Operations

of Mathematical Morphology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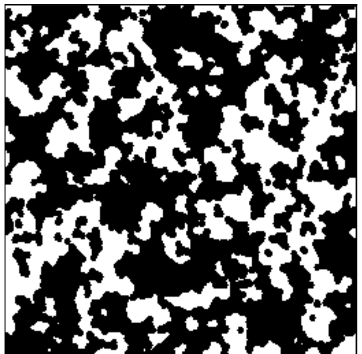
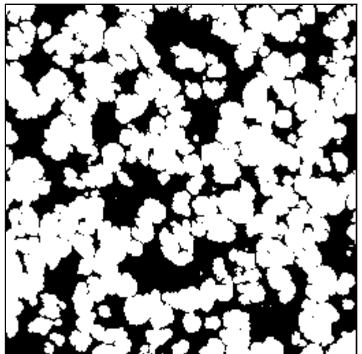

Hexagonal erosion (2)

Basic Operations

of Mathematical Morphology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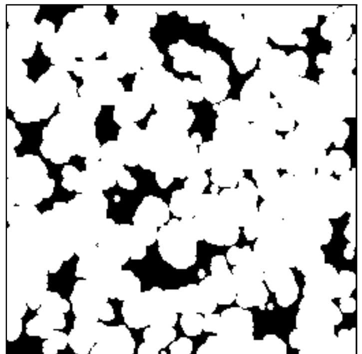

Hexagonal Dilation (2)

Basic Operations

of Mathematical Morphology

Dilation and erosion are not independent operations: to dilate the set A by K is equivalent to erode A^c by K and to take the complement: the two transformations are dual with respect to the complement. We have:

$$
\begin{aligned}
(A \ominus \check{K}_1) \ominus \check{K}_2 = A \ominus (\check{K}_1 \oplus \check{K}_2) \\
(A \cap B) \ominus \check{K} = (A \ominus \check{K}) \cap (B \ominus \check{K}) \\
A \oplus (\check{K}_1 \cup \check{K}_2) = (A \oplus \check{K}_1) \cup (A \oplus \check{K}_2)
\end{aligned} \tag{15}
$$

If A is convex , $A \ominus \check{K}$ is convex. If A and K are convex, $A \oplus \check{K}$ is convex.

Theoretical characterization of a random structure

Characterization of a random set model

Models derived from the theory of **Random Sets** by **G. MATHERON.**

For a **random closed set** A **(RACS)**, characterization by the **CHOQUET capacity** $T(K)$ defined on the compact sets K

$$T(K) = P\{K \cap A \neq \emptyset\} = 1 - P\{K \subset A^c\} = 1 - Q(K)$$

Morphological interpretation

In the euclidean space R^n, translation K_x of the compact set K :

$$K_x = \{x + y; y \in K\}$$

CHOQUET capacity and dilation $\oplus$

$$T(K) = P\{K \cap A \neq \emptyset\} = P\{O \in A \oplus \check{K}\}$$

$$T(K_x) = P\{K_x \cap A \neq \emptyset\} = P\{x \in A \oplus \check{K}\}$$

Functional $Q(K)$ and erosion $\ominus$

$$Q(K_x) = P\{K_x \subset A\} = P\{x \in A^c \ominus \check{K}\}$$

Morphological interpretation

Experimental estimation of $T(K)$ by image analysis, using realizations of A, and dilation operation.

- General case: several realizations and estimation of a frequency for every point x
- For a **stationary** random set, $T(K_x) = T(K)$;
- For an **ergodic** random set, $T(K)$ estimated from a single realization

$$T(K)^* = P\{x \in A \oplus \check{K}\}^* = V_V(A \oplus \check{K})^*$$

Morphological interpretation

Every compact set K brings its own information on the set A

- $K = \{x\}$

$$T(x) = P\{x \in A\}$$

In R^3 $T(x) = p = V_V(A)$

- $K = \{x, x+h\}$

$$T(x, x+h) = P\{x \in A \cup A_{-h}\}$$

$$Q(x, x+h) = P\{x \in A^c \cap A^c_{-h}\}$$

$Q(x, x+h)$ is the covariance of A^c. It depends only on h for a stationary random set.

Useful to study the spatial distribution of A, to measure its scale (correlation length, integral range) or its anisotropy.

Morphological interpretation

Spatial law of A

- $K = \{x_1, x_2, ..., x_n\}$

$$T(K) = 1 - P\{x_1 \in A^c, x_2 \in A^c, ..., x_n \in A^c\}$$

- The spatial law cannot completely characterize the RACS A: $T(K) = 0$ for a stationary point process...
- The spatial law can be used to estimate bounds of the effective physical properties

Morphological interpretation: distance function

$K = B(r)$ (closed ball with radius r, undenumberable set of points)

$$T(B_x(r)) = P\{x \in A \oplus B(r)\}$$

$T(B_x(r))$ enables to estimate the distribution of the random variable $R(x, A)$

$$R(x, A) = \vee\{r; B_x(r) \subset A^c\}$$

$$F_x(r) = P\{R(x, A) \leq r \mid x \in A^c\} = \frac{T(B_x(r)) - T(x)}{1 - T(x)}$$

Useful to study the spatial distribution of A.

Fractal random set A (with a non integer dimension d), $F(r)$ behaves like r^β for $r \to 0$ with $d = n - \beta$.

Random Functions and Random Sets

For a **Random Function** $Z(x)$,

- Continuous function too restrictive.
- Semi-continuous (upper, lower) for which the changes of supports by $\vee$ or by $\wedge$ provide random variables:

$$Z_\vee(K) = \vee_{x\in K}\{Z(x)\}$$

$$Z_\wedge(K) = \wedge_{x\in K}\{Z(x)\}$$

Applications to **Fracture Statistics Models** based on the weakest link assumption, and to the statistics of extremes.

Construction of Random Structure Models

Main steps:

- Choice of basic assumptions
- Computation of the functional $T(K)$

Determination of the functional $T(K)$ as a function of

- the assumptions
- the parameters of the model
- the compact set K.

Construction of Random Structure Models

Calculation of the CHOQUET capacity

For a given model, the functional T is obtained:

- by theoretical calculation
- by estimation
  - on simulations
  - on real structures (possible estimation of the parameters from the "experimental" T , and tests of the validity of assumption).

Construction of Random Structure Models

Calculation of the CHOQUET capacity

- The functions $T(K)$ (K being variable) are **coherent** (which is not the case of any prior analytical model)
- After specification and validation of the model from available data, possible **predictive implementation** of its properties (such as $T(K)$ for compacts K not used during the identification step). Examples: 3D properties deduced from 2D observations (stereology); change of support by $\vee$ or $\wedge$ in the case of a change of scale in fracture statistics.

General properties of the proposed models

- Most random structure models defined in the **Euclidean space** R^n :
  - more general than stochastic processes limited to the 1D space R, where the order relation is used;
  - different from discrete models defined on a grid, even if the discrete counterpart of the euclidean models is easily defined.
- Models depending on a **few number of parameters**, not to ask too much from the available data, and for realistic experimental identification and test.

Morphological Criteria

Granulometry

Size Distribution

- diameters of de cavities
- crack lengths

Access from 1D or 2D slices (spherical shape, or at least convex)

Size Distribution of any media (including connected networks) by **morphological opening** (erosion followed by a dilation) by convex structuring elements

Granulometry

Axiomatic of granulometries

Definition (G. Matheron) A granulometry is a family of set transformations Φ_λ depending on a positive parameter λ (the size), satisfying the following properties:

i) $\Phi_\lambda(A) \subset A$ (Φ_λ is **anti-extensive**)

ii) if $A \subset B, \Phi_\lambda(A) \subset \Phi_\lambda(B)$ (Φ_λ is **increasing**)

iii) $\Phi_\lambda \circ \Phi_\mu = \Phi_\mu \circ \Phi_\lambda = \Phi_{\lambda \vee \mu}$ (**absorption for the composition**)

As a consequence of axiom iii), applied to $\lambda = \mu$, the transformation Φ_λ must be **idempotent:**

$$\Phi_\lambda \circ \Phi_\lambda = \Phi_\lambda$$

Granulometry

Size distribution by opening by convex sets

Openings of the set A by λK, $(\lambda > 0)$, for a **convex set** K, noted K_λ:

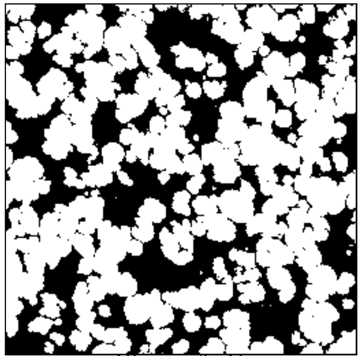
$$\Phi_\lambda(A) = (A \ominus \check{K}_\lambda) \oplus K_\lambda$$

Transformation to be applied to any set (isolated particles, or connected medium):

$$\Phi_\lambda(A) = A_{K_\lambda} = \{x \in A; \exists y \in A \text{ with } K_{\lambda,y} \subset A\} \qquad (21)$$

A_{K_λ} is the set of points of A covered by K_λ translated in space, while remaining included in A.

Granulometry



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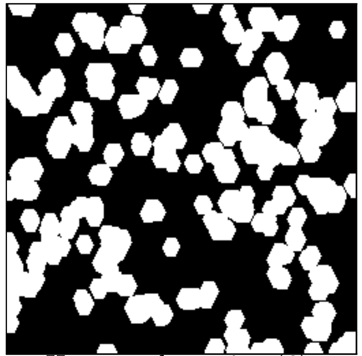


Hexagonal opening (6)

Granulometry

Size distribution by closing by convex sets

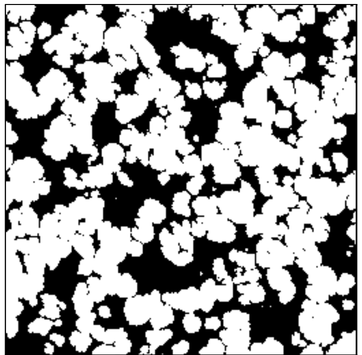
The closing operation is dual of the opening:

$$A^{K_\lambda} = (A \oplus \check{K}_\lambda) \ominus K_\lambda = (\Phi_\lambda(A^c))^c \qquad (22)$$

By closing A by compact convex sets K, size distribution of the set A^c.

The size distribution for a given granulometry is obtained from measures: counting numbers,or measure of a volume in the space R^n.

Granulometry



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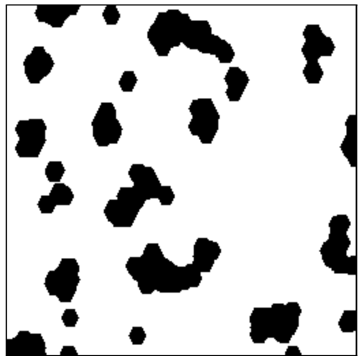


Hexagonal closing (6)

Two dimensional Granulometry

Openings or closings, followed by the measurement of an area (or volume for extension to 3D)

Cumulative measure distribution, from openings by convex structuring elements:

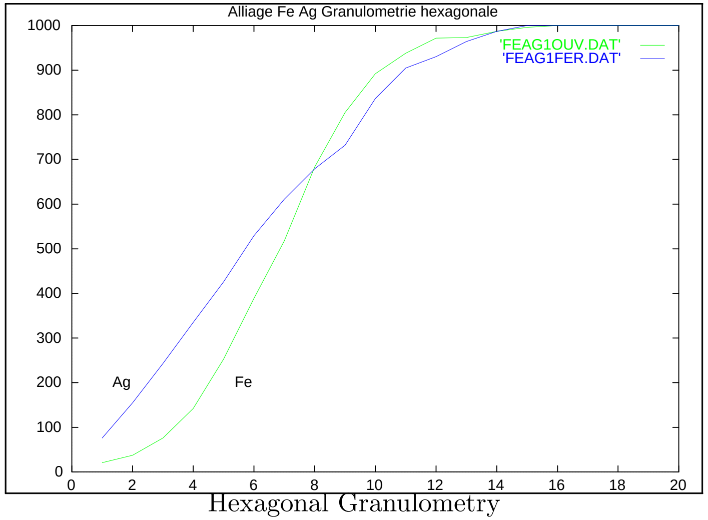
$$G(\lambda) = \frac{P\{x \in A\} - P\{x \in A_{K_\lambda}\}}{P\{x \in A\}} \quad (29)$$

From equation (29) are deduced in R^2 the moments of the area S of the largest K containing x and included in A. For a disc with radius r:

$$E\{S\} = 2\pi \int_0^\infty (1 - G(r))r\, dr$$

$$(30)$$

$$E\{S^n\} = 2n\pi \int_0^\infty (1 - G(r))r^{2n-1}\, dr$$



Size distribution of objects

For isolated objects, size distribution obtained from the distribution of a measurement (area,diameter) made on every object (and correction of edge effects)

Size distribution of spheres

Estimation of the distribution of diameters of a population of spheres in R^3 from data obtained on sections: basic stereological problem ("**unfolding problem**")

Sometimes used for estimating a size distribution of objects with a non spherical shape (grains of a polycrystal seen in section)

Basic relations between properties in R^3 (number of spheres $N_V^{(3)}$ and distribution of diameters in number $F_3(D)$) and induced properties in R^2 (average number of discs $N_V^{(2)}$, distribution of diameters in number $F_2(D)$) and in R (average number of chords $N_V^{(1)}$ and distribution of lengths in number $F_1(D)$)

Size distribution of spheres

From R to R^3:

$$N_V^{(3)} = \frac{2}{\pi} N_V^{(1)} F_1''(0) \tag{31}$$

$$1 - F_3(D) = \frac{1}{D} \frac{F_1'(D)}{F_1''(0)}$$

From dimension i to $i+1$ ($i = 1, 2$):

$$N_V^{(i+1)} = \frac{N_V^{(i)}}{\pi} \int_0^\infty \frac{F_i'(h)}{h}\, dh \tag{32}$$

$$N_V^{(i+1)}(1 - F_{i+1}(D)) = \frac{N_V^{(i)}}{\pi} \int_D^\infty \frac{F_i'(h)}{\sqrt{h^2 - D^2}}\, dh$$

Specific numerical techniques

Spatial Arrangement

Scales, clusters, or preferential associations between component of a microstructure:

- **covariance**
- **distance function**
- **anisotropy**

Spatial Agencement: Covariance

Covariance $C(x, x+h)$ of a random set A

$$C(x, x+h) = P\{x \in A, x+h \in A\} \tag{33}$$

For a **stationary** random set, $C(x, x+h) = C(h)$

If in addition A is **ergodic**, $C(h)$ is estimated from the volume fraction of $A \cap A_{-h}$:

$$C(h) = V_V(A \cap A_{-h}) = V_V(A \ominus \check{h}) \tag{34}$$

Spatial Agencement: Covariance

Estimation of the covariance from images (like plane sections) inside a mask X, by means of the **geometrical covariograms** of the sets $A \cap X$ ($K_{A\cap X}(h)$) and X ($K_X(h)$):

$$C^*(h) = \frac{A((A \cap X) \cap (A \cap X)_{-h})}{A(X \cap X_{-h})} = \frac{K_{A\cap X}(h)}{K_X(h)} \tag{35}$$

Spatial Agencement: Covariance

Result of the erosion by $\{x, x+h\}$, which depends on vector h (by its modulus $\mid h \mid$ and its orientation α) characteristic of the size and of the arrangement of connected objects building the set $A \rightarrow$ variations of $C(h)$ with h

Covariance $Q(h)$ of A^c (with $Q(0) = q = 1 - p$):

$$Q(h) = P\{x \in A^c, x+h \in A^c\} = 1 - 2C(0) + C(h) \qquad (36)$$

The covariance characterizes simultaneously the two sets (A, A^c), while the two granulometries of (A, A^c) bring additional information

Spatial Agencement: Covariance

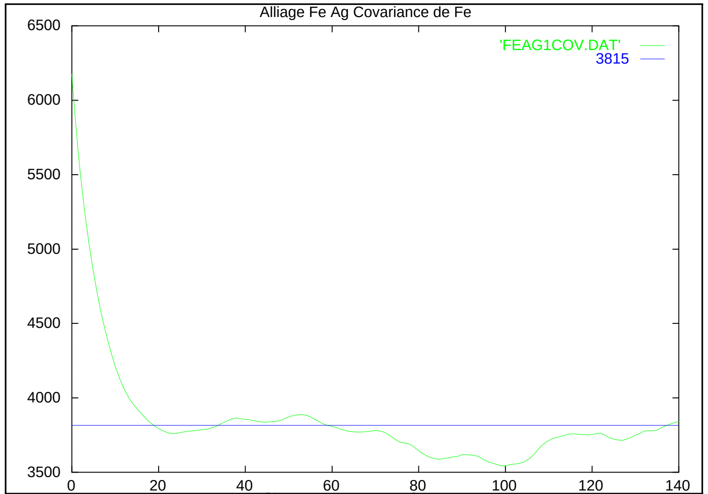
Properties of the covariance of a random set in R^3

- $C(0) = P\{x \in A\} = p$
- $\frac{1}{\pi}\int_0^{4\pi} -\left(\frac{\partial C(h,\alpha)}{\partial h}\right)_{h=0} d\alpha = S_V(A)$ when the partial derivative remains finite.
- If $C(0) - C(h) \simeq h^\beta$ for $h \to 0$, with $0 < \beta < 1$, the boundary of A has a **non integer Haussdorf dimension** $d = 3 - \beta$ (A is a **fractal** set)
- $C(\infty) = p^2$ (the covariance of a stationary and ergodic random set reaches a sill).

Spatial Agencement: Covariance

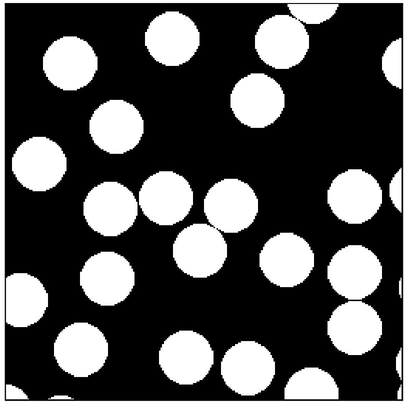
Properties of the covariance of a random set in R^3

- For the orientation α, $C(h)$ reaches a sill at the distance a_α, or **range** (characteristic length scale of the structure): $C(a_\alpha) = C(\infty) = V_V(A)^2 = p^2$
- Presence of various scales $\rightarrow$ inflections of the experimental covariance (nested structures, like clusters, clusters of clusters, etc.)
- Periodicity in images $\rightarrow$ periodicity of the covariance
- Anisotropic structures studied by roses of directions, from the derivative of the covariance in $h = 0$

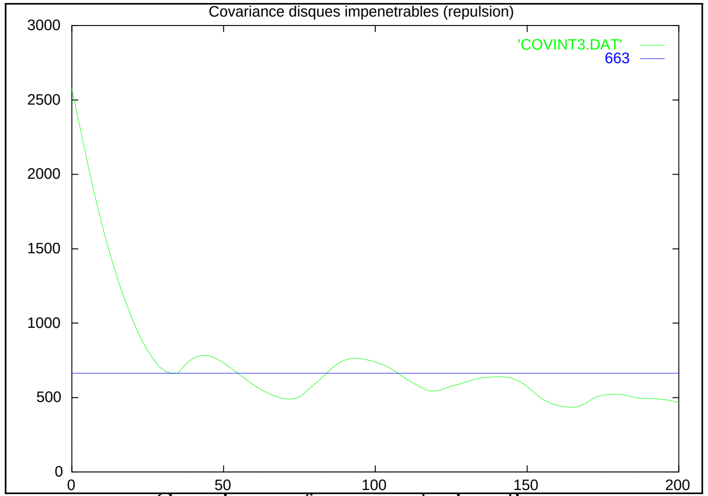


Covariance of Fe

Spatial Agencement: Covariance



Non overlapping random discs



Covariance of non overlapping discs

Spatial Agencement: Covariance

To study physical properties, use of the correlation functions

Centered second order correlation function deduced from the covariance. For a two phase composite with properties $Z = Z_1$ when $x \in A$ and $Z = Z_2$ when $x \in A^c$:

$$\overline{W}_2(h) = E\{(Z(x+h) - E(Z))(Z(x) - E(Z))\}$$
$$= (Z_1 - Z_2)^2 (C(h) - p^2) = (Z_1 - Z_2)^2 (Q(h) - q^2) \quad (38)$$

Spatial Agencement: Cross Covariances

Multicomponent random set with components A_i
$(i = 1, 2, ..., m)$

Separate study for every covariance $C_{ii}(h)$, and mutual associations from the cross covariances $C_{ij}(h)$:

$$C_{ij}(x, x+h) = P\{x \in A_i, x+h \in A_j\} \quad (39)$$

Stationary and ergodic multicomponent random set:

$$C_{ij}(h) = V_V(A_i \cap A_{j-h}) \quad (40)$$

Spatial arrangement from distance distributions

For a random set A and a ball with radius r, $B(r)$:

$$T(B_x(r)) = P\{x \in A \oplus B(r)\}$$

$T(B_x(r))$ allows us to estimate the distribution of the random variable $R(x, A)$, or distribution of the first point of contact, where the symbol $\vee$ means the upper bound (sup):

$$R(x, A) = \vee\{r; B_x(r) \subset A^c\}$$

$$F_x(r) = P\{R(x, A) \leq r \mid x \in A^c\} = \frac{T(B_x(r)) - T(x)}{1 - T(x)}$$

Spatial arrangement from distance distributions

For a stationary random set A,

$$T(r) = 1 - F(r) = \frac{1 - P\{x \in A \oplus B(r)\}}{1 - p} \quad (44)$$

The moments of $F(r)$ allow us to summarize this distribution.

For a **fractal** random set A (with irregular boundaries with non integer dimension d), $F(r)$ behaves as r^β when $r \to 0$ with $d = n - \beta$

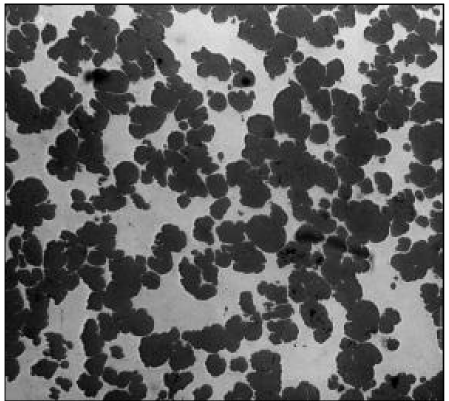


Figure 1

Figure 1: Image of a two-phase alloy Fe (black) Ag (grey).

Spatial arrangement from distance distributions

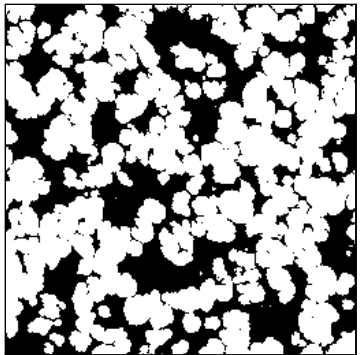
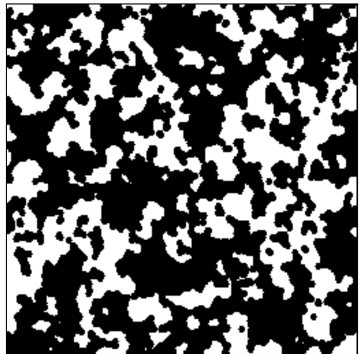


Figure 2: Fe



Hexagonal erosion (2)

Spatial arrangement from distance distributions

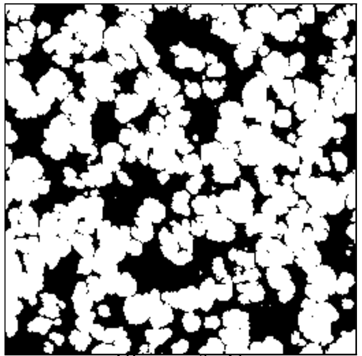
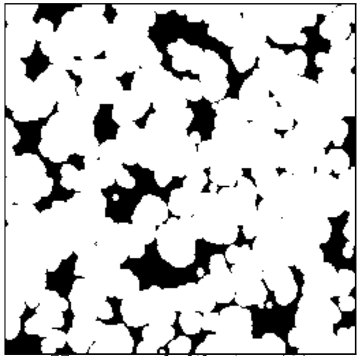
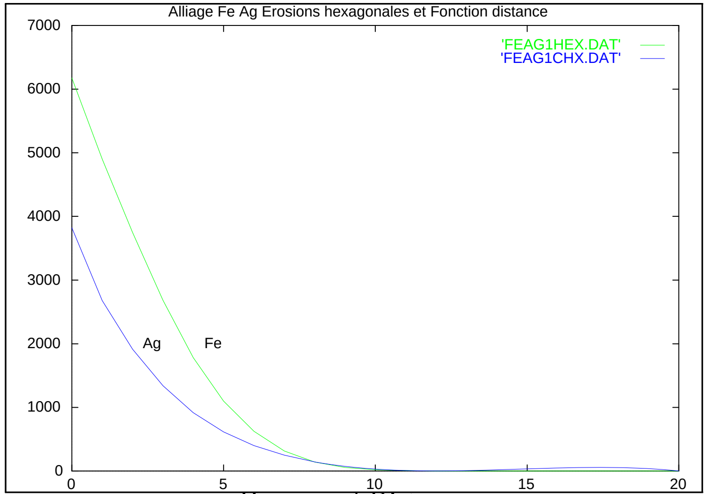


Figure 2: Fe



Hexagonal dilation (2)



Hexagonal Distance

Morphological criteria

Spatial arrangement: Anisotropy

- **Directional Measurements** : variation with orientation of the covariance or of the histogram of chord lengths
- **Roses of orientation**, obtained by counting in the direct space or by directional filtering in the Fourier plane → characterization of damage with respect to a coordinate system (orientation off the applied stress, or crystallographic orientation)

Conclusion

Morphological Characterization of a microstructure → **great number of parameters** according to various criteria

Every type of measurement → specific aspect of the structure

- **Synthesis from probabilistic models**
- Use of these morphological measurements to **predict the macroscopic behavior** of materials (change of scale models)