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Image decomposition by variational models

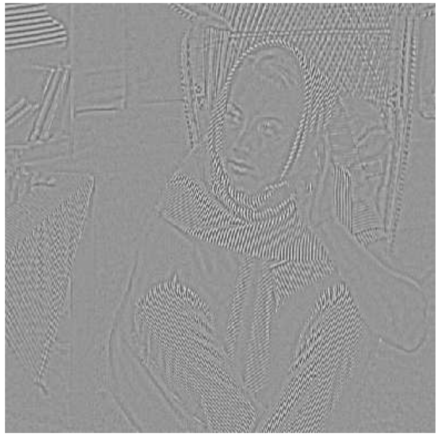
Workshop on Models and Images for Porous Media

Paris, January 12-16, 2009

$u+v$ decomposition :

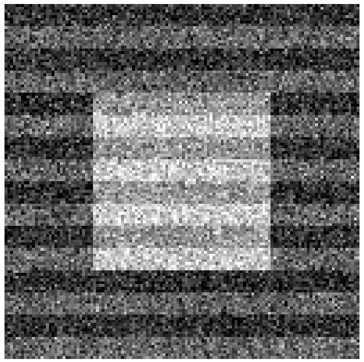
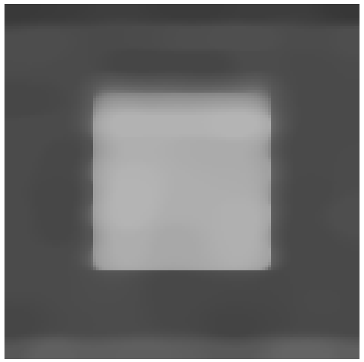
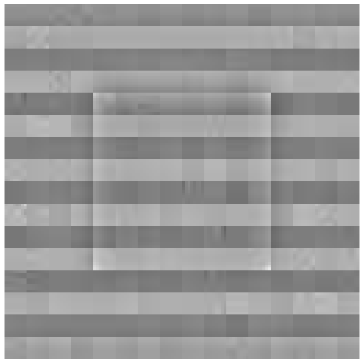
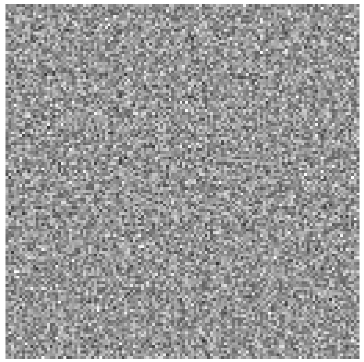
Initial image (f) $=$ BV (u) $+$ oscillatory component (v)





$u+v+w$ decomposition :

Initial image (f) $=$ geometry (u) $+$ texture (v) $+$ noise (w)

Overview

1) Problem statement

2) $TV + G$ model

3) $TV + G + E$ model

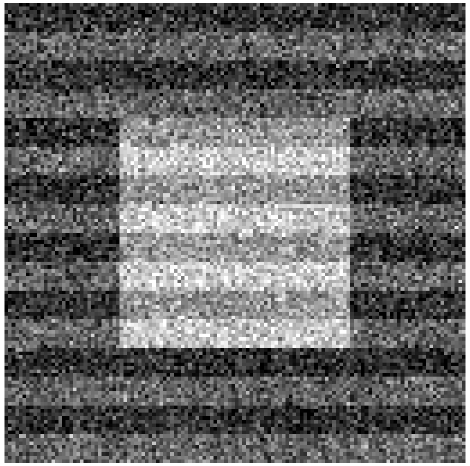
4) TV-Gabor model

5) Parameter selection

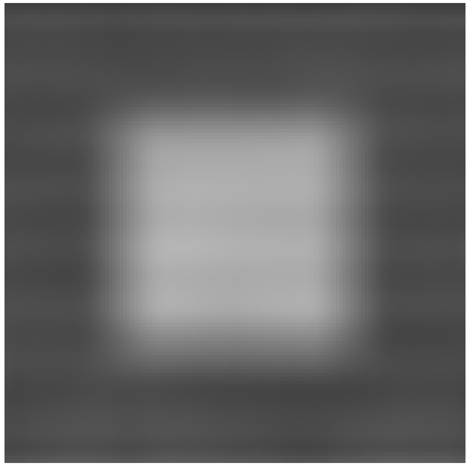
6) $TV - L^1$ model

Linear filtering

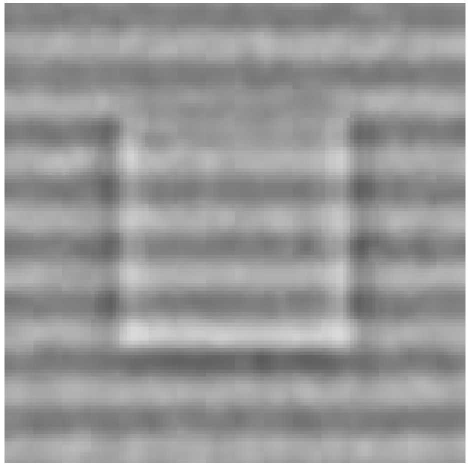
Noisy image ($\sigma = 35$)



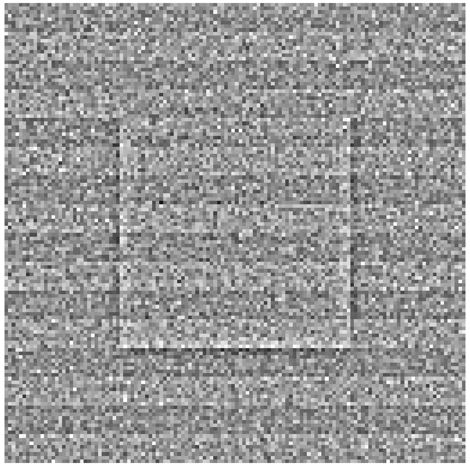
u (low frequencies)



v (medium frequencies)



w (high frequencies)



Rudin-Osher-Fatemi model

(Physica D. 1992)

Problem of image restoration (f degraded image, u restored image) :

$$f = \mathcal{R}u + n$$

A way to reconstruct u :

$$\inf_u \underbrace{\frac{1}{2\lambda}\|f - \mathcal{R}u\|_2^2}_{\text{data term}} + \underbrace{L(u)}_{\text{regularization}}$$

Here we will assume that $\mathcal{R} = Id$.

In the ROF model, one uses $L(u) = J(u)$ with :

$$J(u) = \int_\Omega |Du| = \sup\left\{\int_\Omega u(x)\mathrm{div}\,(\xi(x))dx / \xi \in C_c^\infty(\Omega, \mathbb{R}^N), \|\xi\|_{L^\infty(\Omega,\mathbb{R}^N)} \leq 1\right\}$$

If u is regular, then $J(u) = \int_\Omega |\nabla u|\, dx$.

Rudin-Osher-Fatemi model

Setting $v = f - u$, the problem can be rewritten :

$$\inf_{(u,v)\in BV\times L^2/f=u+v}\left(J(u) + \frac{1}{2\lambda}\|v\|_2^2\right)$$

But the L^2 norm does not measure oscillations.

Example : $\Omega = (0, 2\Pi)$, and $f_n(x) = \cos(nx)$.

The larger n, the more oscillatory f_n is.
$\|f_n\|^2_{L^2(\Omega)} = \frac{1}{2}$.

$\Longrightarrow$ Need to choose another norm which goes to 0 when f_n oscillates.

Meyer's model

Y. Meyer (2001) has proposed the following model :

$$\inf_{(u,v)\in BV\times G/f=u+v} \left(J(u)+\alpha\|v\|_G\right)$$

The Banach space G contains signals with strong oscillations, and thus in particular textures and noise.

Definition : G is the Banach space composed of generalized functions v which can be written

$$v = \partial_1 g_1 + \partial_2 g_2 = \operatorname{div}(g)$$

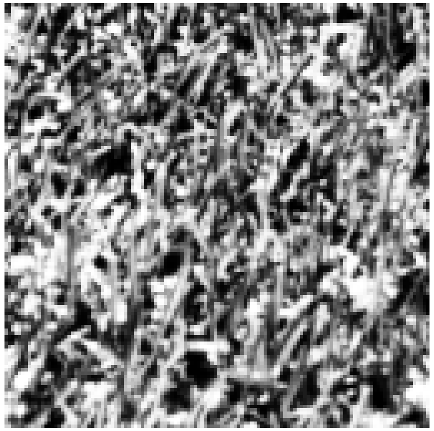
with g_1 and g_2 in L^∞.

$$\|v\|_G = \inf\left\{\|g\|_\infty / v = \operatorname{div}(g), g = (g_1, g_2),\ g_1 \text{ et } g_2 \in L^\infty, |g(x)| = \sqrt{|g_1|^2 + |g_2|^2}(x)\right\}$$

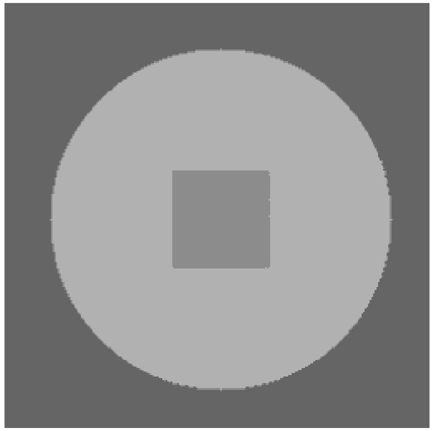
Example : $\Omega = (0, 2\Pi)$, and $f_n(x) = \cos(nx) = \left(\frac{1}{n}\sin(nx)\right)'$.
Then $\|f_n\|_G \leq \frac{1}{n}$.

Example

Textured image



Geometrical image



Images	TV	L^2	G	TV/G
Textured image	1 000 000	9 500	360	86
Geometrical image	64 600	9 500	2 000	1

Remak : (G. Strang 1982)

$$\|f\|_{G(\Omega)} = \sup_{E \subset \Omega} \frac{\int_E f}{P(E, \Omega)}$$

$\Longrightarrow$ Notion of scale (Strong-Aujol-Chan, MMS 2006)

Some models

We use the notation :

$$J(u) = \int |Du|$$

$TV - L^2$ model (Rudin-Osher-Fatemi) :

$$\inf_{(u,v) \in BV \times L^2 / f = u+v} \left(J(u) + \lambda \|v\|_{L^2}^2 \right)$$

$TV - G$ model (Meyer) :

$$\inf_{(u,v) \in BV \times G / f = u+v} (J(u) + \lambda \|v\|_G)$$

$TV - E$ model (Meyer) (E Besov space) :

$$\inf_{(u,v) \in BV \times G / f = u+v} (J(u) + \lambda \|v\|_E)$$

$TV - F$ model (Meyer) (F=div(BMO) space) :

$$\inf_{(u,v) \in BV \times G / f = u+v} (J(u) + \lambda \|v\|_F)$$

Some other models

$TV - H^{-1}$ model (Osher-Sole-Vese)

$$\inf_{(u \times v) \in BV \times \mathcal{H} / f = u+v} \left(J(u) + \lambda \|v\|^2_{H^{-1}} \right)$$

$TV - W^{-1,p}$ model (Vese et al)

$$\inf_{(u \times v) \in BV \times \mathcal{H} / f = u+v} \left(J(u) + \lambda \|v\|_{W^{-1,p}} \right)$$

TV-Hilbert model (Aujol-Gilboa)

$$\inf_{(u \times v) \in BV \times \mathcal{H} / f = u+v} \left(J(u) + \lambda \|v\|^2_{\mathcal{H}} \right)$$

$TV - L^1$ model (Nikolova) :

$$\inf_{(u,v) \in BV \times L^1 / f = u+v} \left(J(u) + \lambda \|v\|_{L^1} \right)$$

Sparse-based approaches (Starck-Elad-Donoho, Daubechies-Teschke, ...).

Overview

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Discretisation

An image is a two-dimensional vector of size $N \times N$.

We denote by X the Euclidean space $\mathbb{R}^{N\times N}$, and $Y = X \times X$. X is embeded with the Euclidean scalar product : $(u,v)_X = \sum_{1\leq i,j\leq N} u_{i,j}v_{i,j}$ and the norm : $\|u\|_X = \sqrt{(u,u)_X}$.

If $u \in X$, than ∇u is a vector in Y given by :

$(\nabla u)_{i,j} = ((\nabla u)^1_{i,j}, (\nabla u)^2_{i,j})$

The discrete total variation of u is then given by :

$$J(u) = \sum_{1\leq i,j\leq N} |(\nabla u)_{i,j}|$$

We also introduce a discrete version of the divergence. We define it as in the continuous case :

$$\mathrm{div} = -\nabla^*$$

where ∇^* is the adjoint of ∇ : i.e., for all $p \in Y$ and $u \in X$,

$$(-\mathrm{div}\, p, u)_X = (p, \nabla u)_Y$$

Discrete G space

Definition :

$$G = \{v \in X \ / \ \exists g \in Y \text{ such that } v = \operatorname{div}(g)\}$$

and if $v \in G$:

$$\begin{aligned}\|v\|_G &= \inf \{ \|g\|_\infty \ / \ v = \operatorname{div}(g), \\ &\qquad g = (g^1, g^2) \in Y, |g_{i,j}| = \sqrt{(g^1_{i,j})^2 + (g^2_{i,j})^2} \Big\}\end{aligned}$$

where $\|g\|_\infty = \max_{i,j} |g_{i,j}|$.
Moreover, we denote :

$$G_\mu = \{v \in G \ / \ \|v\|_G \le \mu\}$$

Properties

Proposition :

$$J(u) = \sup_{v \in G_1} (u,v)_X$$

and

$$\|v\|_G = \sup_{J(u) \leq 1} (u,v)_X$$

In particular, one has $J^*(v) = \chi_{G_1}(v)$ where $J^*(v) = \sup \left((u,v)_X - J(u) \right)$ and

$$\chi_{G_1}(v) = \begin{cases} 0 \text{ if } v \in G_1 \\ +\infty \text{ otherwise} \end{cases}$$

Proposition : G can also be written :

$$X_0 = \{ v \in X \;/\; \sum_{i,j} v_{i,j} = 0 \}$$

Chambolle's projection algorithm

A. Chambolle has proposed an efficient algorithm to compute $P_{G_\lambda}(f)$, the orthogonal projection of f on G_λ (MIA 2002, JMIV 2004).
We want to solve the problem :

$$\min \left\{ \|\lambda \mathrm{div}\,(p) - f\|_X^2 \ / \ p \in X \times X \ , \ |p_{i,j}| \leq 1 \ \forall i,j = 1, \dots, N \right\}$$

Algorithm (fixed point) :

$$p^0 = 0$$

$$p_{i,j}^{n+1} = \frac{p_{i,j}^n + \tau(\nabla(\mathrm{div}\,(p^n) - f/\lambda))_{i,j}}{1 + \tau|(\nabla(\mathrm{div}\,(p^n) - f/\lambda))_{i,j}|}$$

Sufficient condition for the algorithm to converge :
Theorem : If $\tau \leq 1/8$, then $\lambda \mathrm{div}\,(p^n)$ converges to $P_{G_\lambda}(f)$ when $n \to +\infty$.

Variant of Chambolle's projection algorithm

$$\min \left\{ \|\lambda \mathrm{div}\,(p) - f\|_X^2 \ / \ p \in X \times X \ , \ |p_{i,j}| \leq 1 \ \forall i,j = 1, \dots, N \right\}$$

Algorithm (projected gradient) : (Chambolle 2004)

$$p^0 = 0$$

$$p_{i,j}^{n+1} = \frac{p_{i,j}^n + \tau (\nabla (\mathrm{div}\,(p^n) - f/\lambda))_{i,j}}{\max \left\{ 1, p_{i,j}^n + \tau |(\nabla (\mathrm{div}\,(p^n) - f/\lambda))_{i,j}| \right\}}$$

It can be proved that (Aujol 2008, Duval et al 2008) :

Theorem : If $\tau \leq 1/4$, then $\lambda \mathrm{div}\,(p^n)$ converges to $P_{G_\lambda}(f)$ when $n \to +\infty$.

In practice, for typical image restoration problem, it appears to be 30% faster than the original projection algorithm.

Remark : Faster but more complicated algorithms (based on Nesterov's schemes) can be used to compute the projection (Weiss et al 2006, Aujol 2008).

Solving ROF

Chambolle's projection algorithm can be used to minimize the total variation (MIA 2002, JMIV 2004).

$$\inf_{u \in BV} \left(J(u) + \frac{1}{2\lambda} \|f - u\|_2^2 \right)$$

Proposition : The solution of the above problem is given by :

$$u = f - P_{G_\lambda}(f)$$

where P is the orthogonal projection on G_λ.

We recall that :

$$G_\lambda = \{ v \in G / \|v\|_G \leq \lambda \}$$

(Aujol et al 2003)

We want to solve :

$$\inf_{(u,v)\in BV\times G_\mu} \left(J(u) + \frac{1}{2\lambda}\|f-u-v\|_2^2 \right)$$

where

$$G_\mu = \{v \in G / \|v\|_G \leq \mu\}$$

The parameter λ controls the L^2 norm of the residual $f-u-v$. The parameter μ controls the $||.||_G$ norm of v.

Remark :

It is a way to approximate the $TV - G$ model :

$$\inf_u J(u) + \alpha\|f-u\|_G$$

Principle

We solve the two following problems :

- v fixed, we solve

$$\inf_{u \in BV} \left(J(u) + \frac{1}{2\lambda} \|f - u - v\|_2^2 \right) \tag{1}$$

- u fixed, we solve

$$\inf_{v \in G_\mu} \|f - u - v\|_2^2 \tag{2}$$

The solution of (1) is given by :

$$\hat{u} = f - v - P_{G_\lambda}(f - v)$$

where P_{G_λ} is the orthogonal projection on G_λ.
The solution of (2) is given by :

$$\hat{v} = P_{G_\mu}(f - u)$$

Example :

Zebra ($\lambda = 1$, $\mu = 50$ and $\mu = 100$)

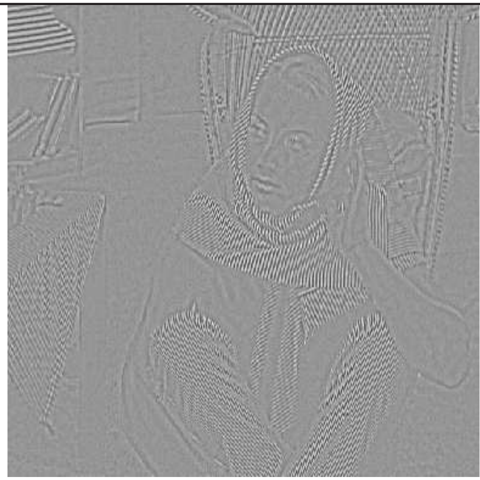
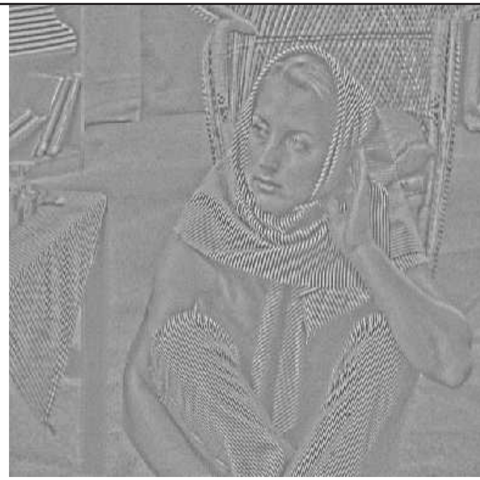
Original image (f)	BV component (u)	BV component (u)
		

Reconstructed image ($u + v$)	$v + 150.0$	$v + 150.0$
		

Comparison with the Rudin-Osher-Fatemi model :

Barbara ($\lambda = 1$ and $\mu = 50$)

Original image (f)	u	u (ROF)
		

Reconstructed image ($u + v$)	($v + 150.0$)	$v + 150.0$ (ROF)
		

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Besov spaces

Definition : $\dot{B}^1_{1,1}$ is the usual homogeneous Besov space. Let $\psi_{j,k}$ an orthonormal basis of smoothed wavelets with compact support, then $\dot{B}^1_{1,1}$ is the subspace of $L^2(\mathbb{R}^2)$ of functions f such that :

$$\sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}^2} |c_{j,k}| < +\infty$$

where the $c_{j,k}$ are the wavelet coefficients of f.

Definition : The dual space of $\dot{B}^1_{1,1}$ is the Banach space $E = \dot{B}^{\infty}_{-1,\infty}$. It is characterized by the fact that the wavelet coefficients of a generalized function in $E = \dot{B}^{\infty}_{-1,\infty}$ are in $l^{\infty}(\mathbb{Z} \times \mathbb{Z}^2)$.

Remark : We have

$$\dot{BV} \subset G \subset E$$

Another decomposition model

(Meyer 2001)

$$\inf_{u+v=f} \left(J(u) + \beta \|v\|_E \right)$$

(Aujol-Chambolle 2004)

$$\inf_{(u,v)\in X^2} \left(J(u) + B^*(v/\delta) + \frac{1}{2\lambda} \|f-u-v\|^2 \right)$$

where

$$B(w) = \|w\|_{\dot{B}^1_{1,1}}$$

and thus

$$B^*(v/\delta) = \chi_{E_\delta}(v)$$

with

$$E_\delta = \{ v \ / \ \|v\|_E \leq \delta \}$$

Convex analysis

Proposition : (Chambolle et al, IEEE TIP 1998)

The solution of the functional :

$$\inf_u \|f-u\|^2 + 2\tau B(u)$$

is given by $u = WST(f,\tau)$, where $WST(f,\tau)$ is the wavelet soft thresholding of f (with threshold value λ).

Proposition :

The two following statements are equivalent :

1. $\tilde{u}$ is a solution of
$$\min_u \left(B(u) + \frac{1}{2\lambda}\|f-u\|_2^2 \right)$$
2. $\tilde{v} = f - \tilde{u}$ is a solution of
$$\min_v \left(B^*(v/\lambda) + \frac{1}{2\lambda}\|f-v\|_2^2 \right)$$

Hence the solution of 2. is given by $\tilde{v} = f - WST(f,\lambda)$.

Minimization

We consider the two following problems :

- v fixed, we compute u as the solution of :

$$\inf_{u \in X} \left(J(u) + \frac{1}{2\lambda} \|f - u - v\|_X^2 \right) \tag{1}$$

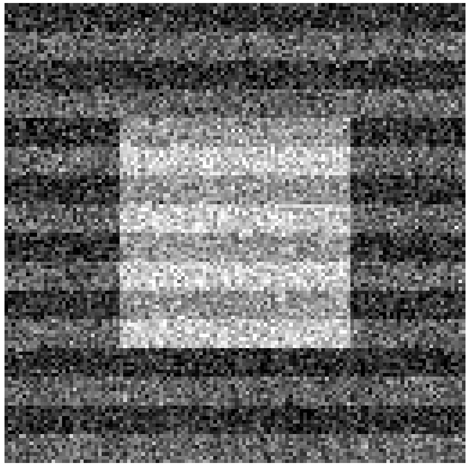
- u fixed, we compute v as the solution of :

$$\inf_{v \in E_\delta} \|f - u - v\|_X^2 \tag{2}$$

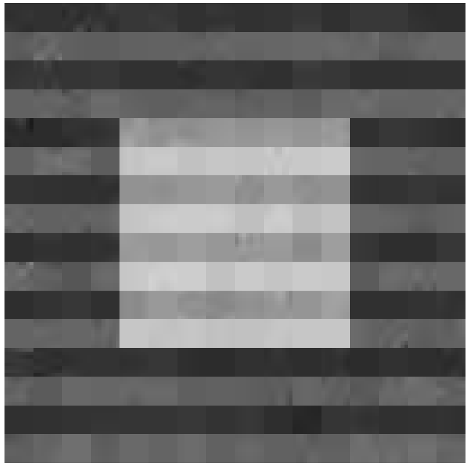
The solution of (1) is given by : $\hat{u} = f - v - P_{G_\lambda}(f - v)$.
And the solution of (2) is given by : $\hat{v} = P_{E_\delta}(f - u) = f - u - WST(f - u, \delta)$, where $WST(f - u, \delta)$ corresponds to the wavelet soft thresholding of $f - u$ (with threshold value δ).

Comparison between the G and the E norm

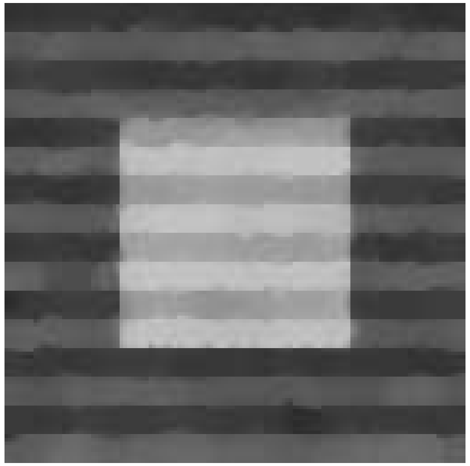
Noisy image f ($\sigma = 35$)



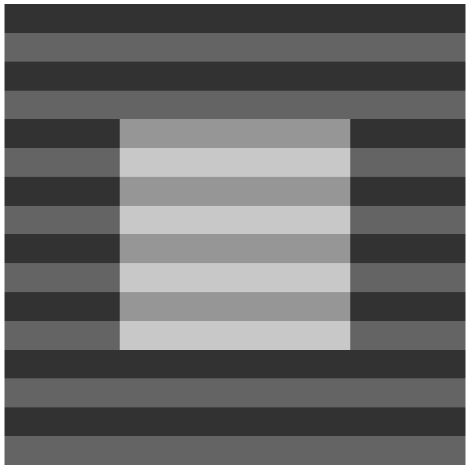
Restored image (E norm)



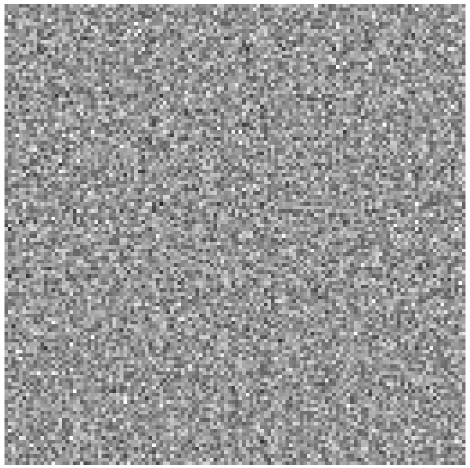
Restored image (G norm)



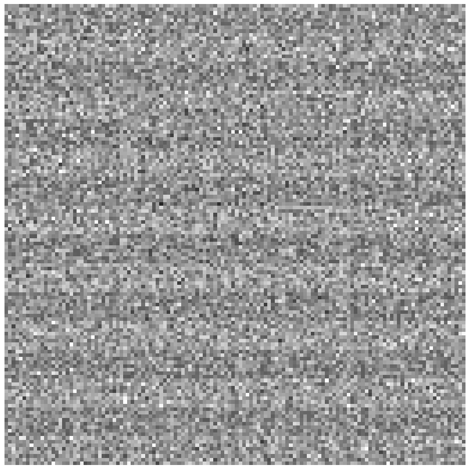
Original image



Noise



Noise



$u + v + w$ model

(Aujol-Chambolle 2004)

We propose to minimize the following functional :

$$\inf_{(u,v,w)\in X^3} F(u,v,w)$$

where

$$F(u,v,w) = J(u) + J^*\left(\frac{v}{\mu}\right) + B^*\left(\frac{w}{\delta}\right) + \frac{1}{2\lambda}\|f-u-v-w\|^2$$

We recall that :

$$J^*(v/\mu) = \chi_{G_\mu}(v)$$

$$B^*(w/\delta) = \chi_{E_\delta}(w)$$

Principle

- v and w fixed, u solution of :

$$\inf_{u \in X} \left(J(u) + \frac{1}{2\lambda} \|f - u - v - w\|_X^2 \right) \tag{1}$$

- u and w fixed, v solution of :

$$\inf_{v \in G_\mu} \|f - u - v - w\|_X^2 \tag{2}$$

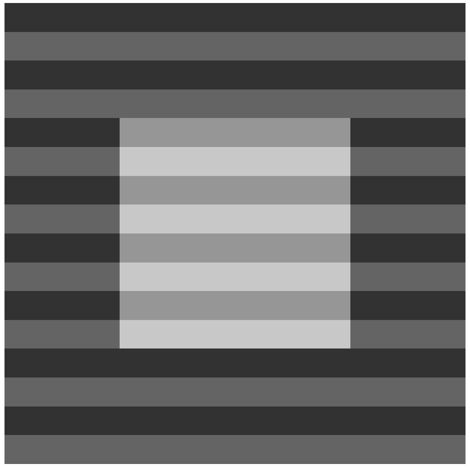
- u and v fixed, w solution of :

$$\inf_{w \in E_\delta} \|f - u - v - w\|_X^2 \tag{3}$$

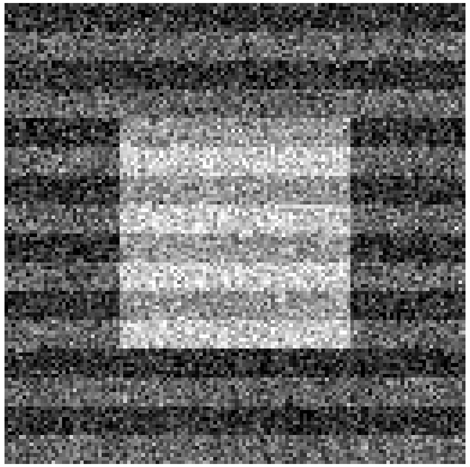
The solution of (1) is given by : $\hat{u} = f - v - w - P_{G_\lambda}(f - v - w)$.
The solution of (2) is given by : $\hat{v} = P_{G_\mu}(f - u - w)$.
The solution of (3) is given by : $\hat{w} = P_{E_\delta}(f - u - v) = f - u - v - WST(f - u - v, \delta)$.

Results

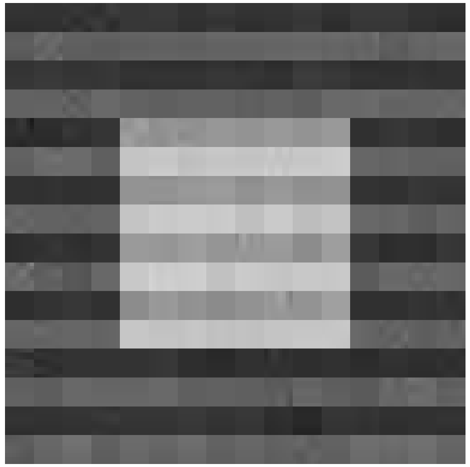
Original image



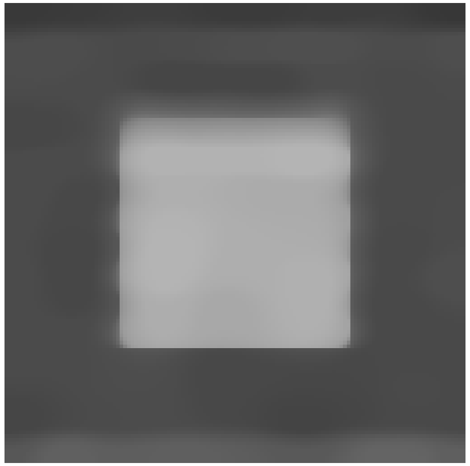
Noisy image ($\sigma = 35$)



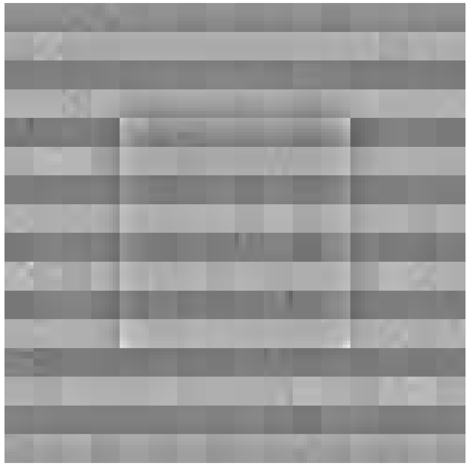
$u + v$



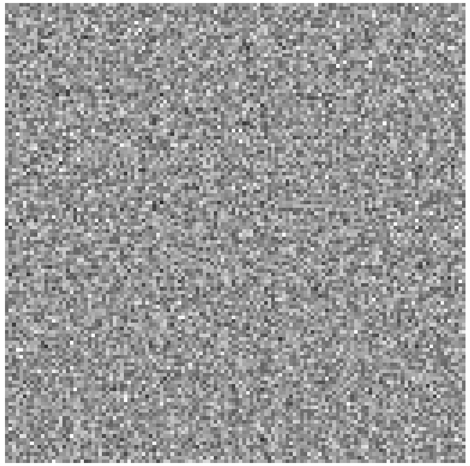
u



$v + 150.0$

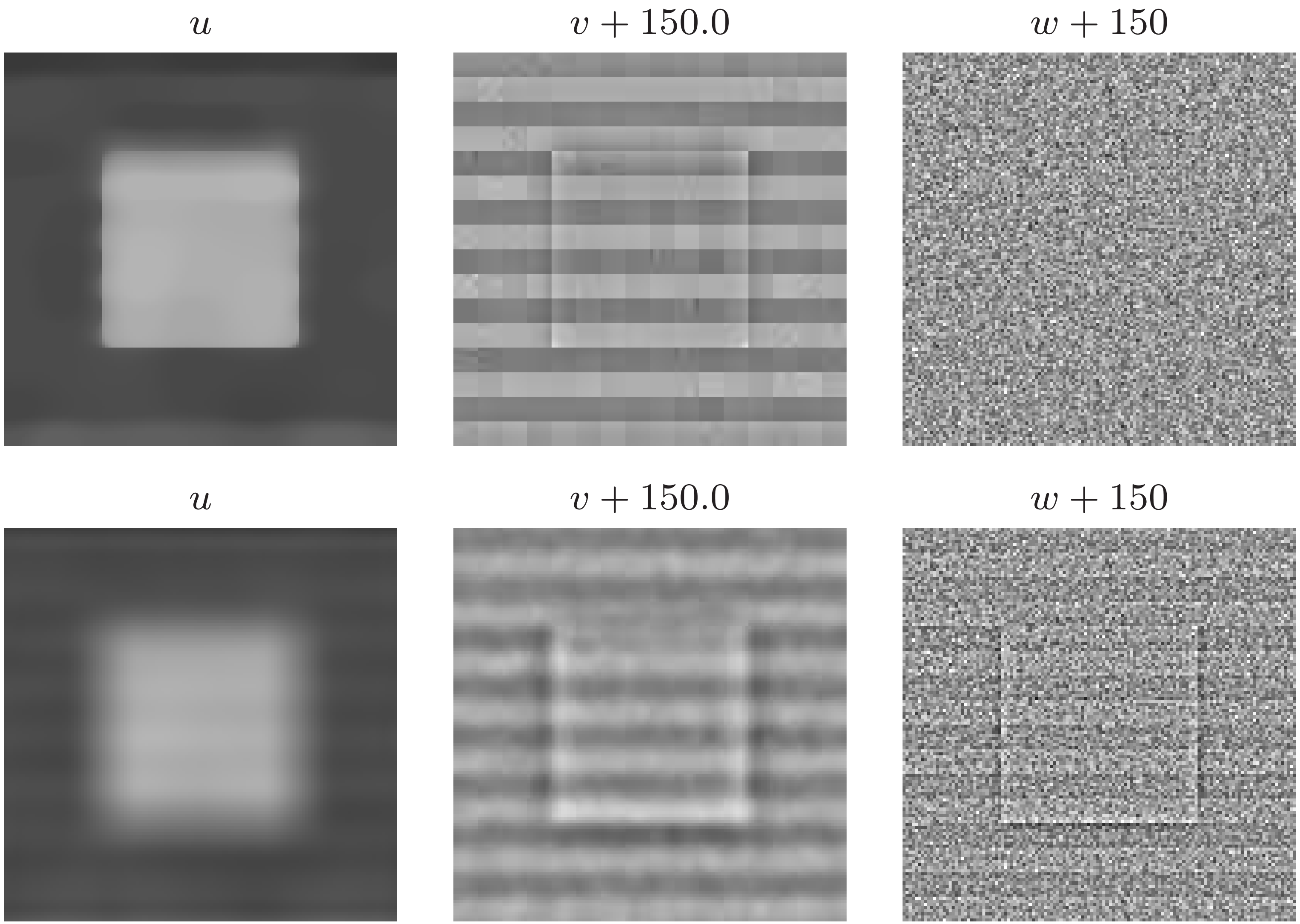


$w + 150$



A simple case ($\lambda = 0.5$, $\mu = 120$, $\eta = 1.0$, Haar)

Comparison



First line : $u + v + w$ algorithm. Second line : Gaussian filtering.

Results

Original image

Noisy image ($\sigma = 20$)

Barbara Image I

Results

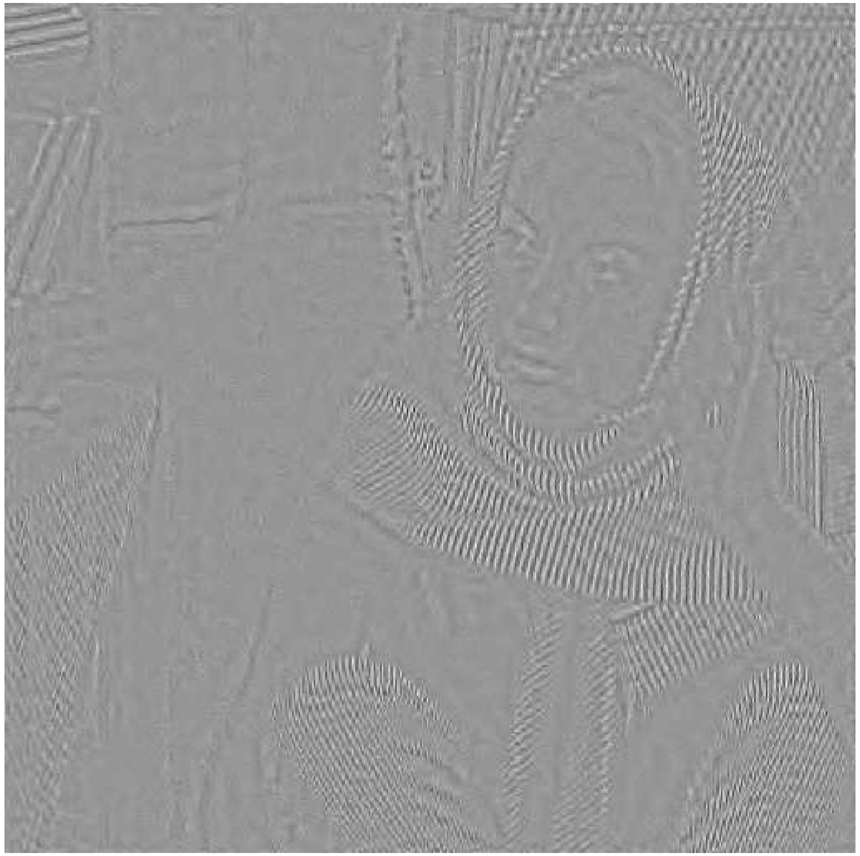
u

$u+v$

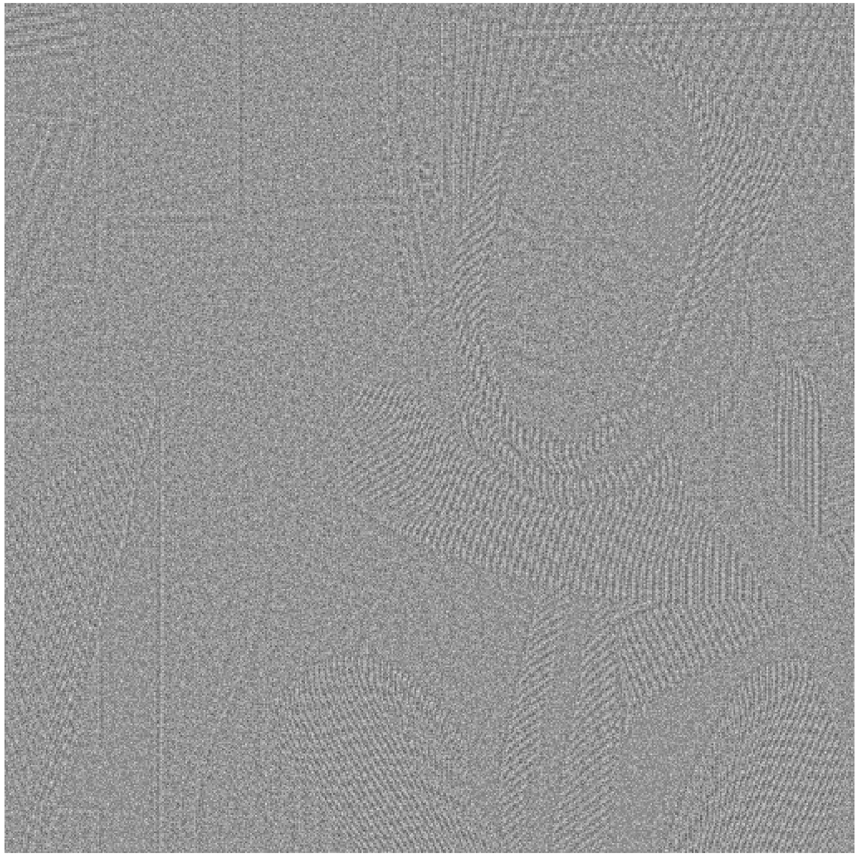
Barbara Image II ($\lambda = 1.0$, $\mu = 30$, $\eta = 0.6$, Daub8)

Results

$v + 150.0$



$w + 150$



Barbara image III ($\lambda = 1.0$, $\mu = 30$, $\eta = 0.6$, Daub8)

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6) $TV - L^1$ model

Negative Sobolev norm

$$H = W^{1,2} = \{f \in L^2 \ / \ \nabla f \in L^2\}$$

$$H^{-1} = W^{-1,2} = \left(W_0^{1,2}\right)'$$

We set :

$$\|u\|_H = \|\nabla u\|_Y = \left(\sum_{1 \leq i,j \leq N} |\nabla u_{i,j}|^2\right)^{1/2}$$

The associated polar semi-norm is :

$$\|v\|_{H^{-1}} = \sup_{\|u\|_H = 1} (v, u)_X$$

One can show that

$$\|u\|_{H^{-1}} = \sqrt{(u, -\Delta^{-1} u)_X}$$

(easy to compute with the discrete Fourier transform)

Osher-Sole-Vese model

Osher-Sole-Vese have introduced the following functional (MMS 2003) :

$$\inf_u \left(J(u) + \frac{1}{2\lambda} \|f - u\|_{H^{-1}}^2 \right)$$

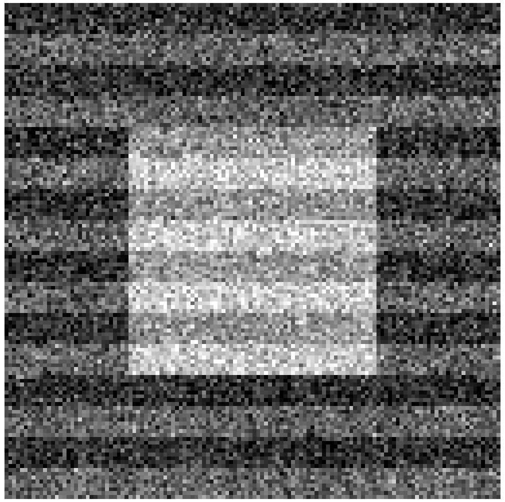
They compute the associated Euler-Lagrange equation, and solve a fourth order PDE.

Remark : No residual in this model.

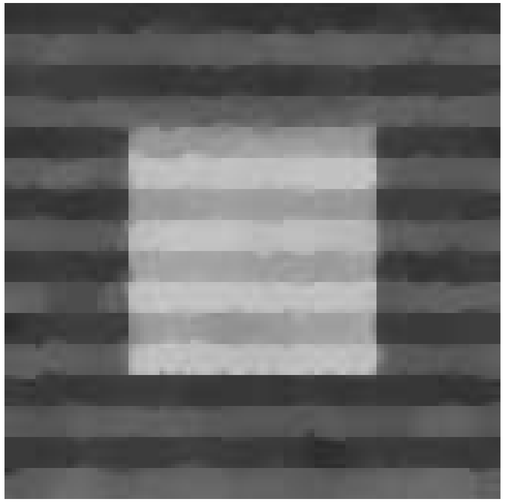
Other approaches : Daubeshie-Teschke 04 (wavelets based algorithm), Aujol-Chambolle 04 (projection algorithm), ...

Denoising

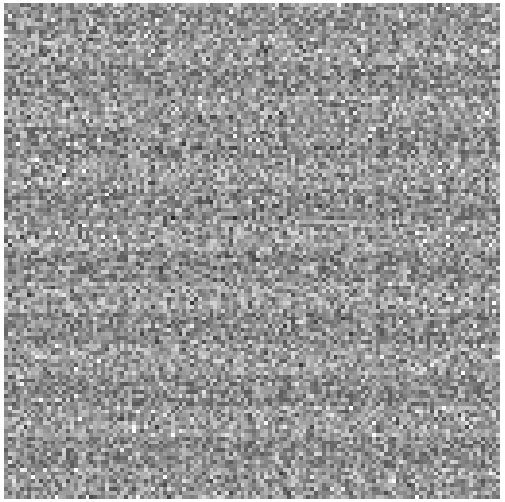
Noisy image f ($\sigma = 35$)



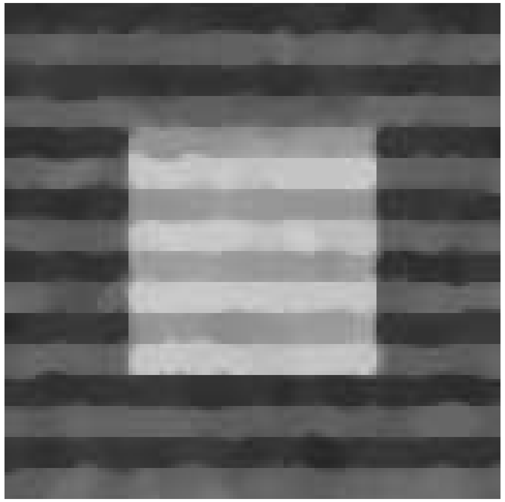
Restored image (G)



Noise (G)



Restored image (OSV)



Noise (OSV)

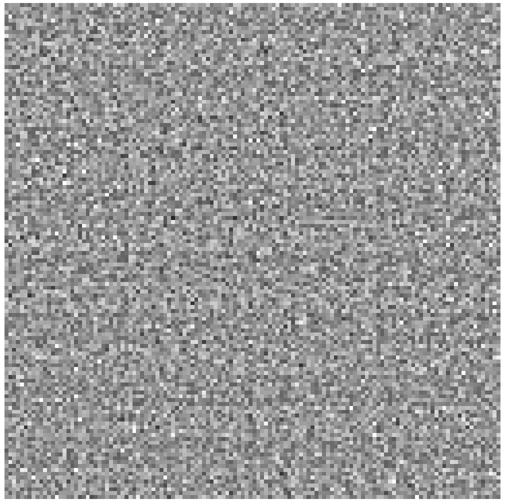
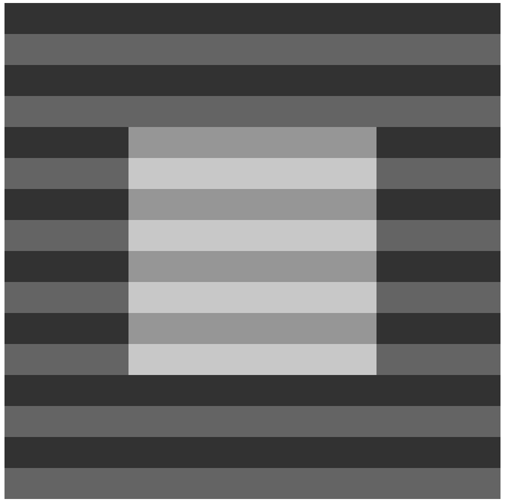
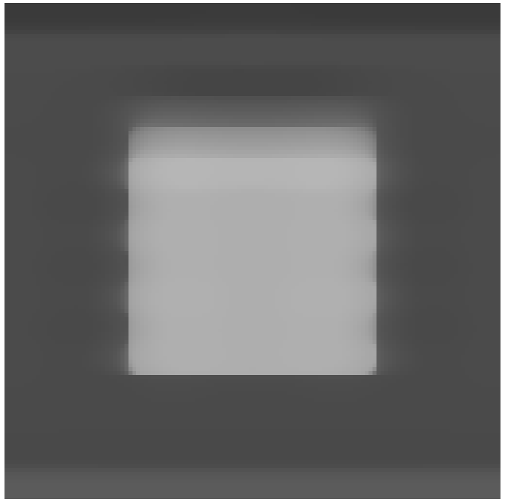


Image decomposition

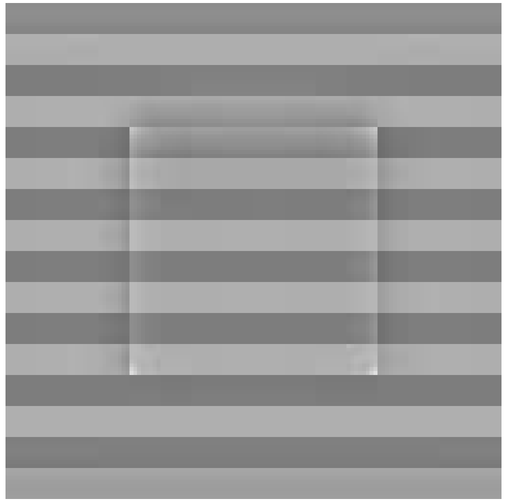
Original image f



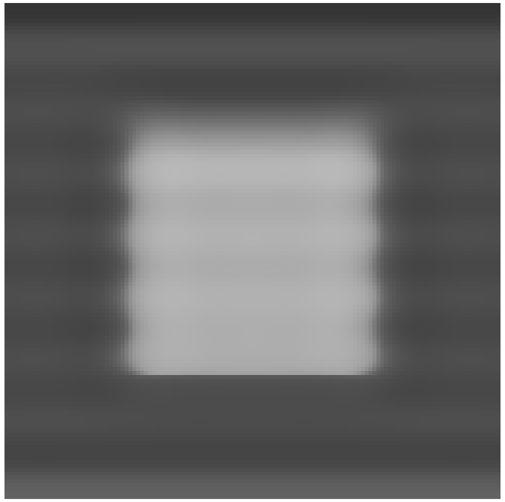
u (G)



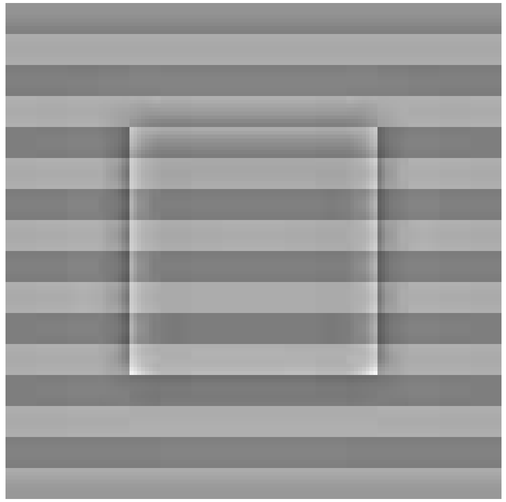
v (G)



u_{OSV}



v_{OSV}



TV-Hilbert

Aujol-Gilboa 2004 :

$$\inf_u \left(J(u) + \frac{\lambda}{2} \|f - u\|_{\mathcal{H}}^2 \right)$$

K is a linear positive symmetric operator, and

$$\langle f, g \rangle_{\mathcal{H}} = \langle f, Kg \rangle_{L^2}$$

1. When $K = Id$, then $\mathcal{H} = L^2 \Longrightarrow$ ROF model
2. When $K = -\Delta^{-1}$, then $\mathcal{H} = H^{-1} \Longrightarrow$ OSV model

Projection algorithm

$$\inf_u \left(J(u) + \frac{\lambda}{2} \|f - u\|^2_{\mathcal{H}} \right) \tag{1}$$

It is possible to adapt Chambolle's projection algorithm to this functional (Aujol-Gilboa 2004).

Algorithm :

$$p^0 = 0$$

and

$$p^{n+1}_{i,j} = \frac{p^n_{i,j} + \tau (\nabla (K^{-1} \mathrm{div}\,(p^n) - \lambda f))_{i,j}}{1 + \tau |(\nabla (K^{-1} \mathrm{div}\,(p^n) - \lambda f))_{i,j}|}$$

Theorem : If $\tau \leq \frac{1}{8\|K^{-1}\|_{L^2}}$, then $\frac{1}{\lambda} K^{-1} \mathrm{div}\, p^n \to \hat{v}$ as $n \to \infty$, and $f - \frac{1}{\lambda} K^{-1} \mathrm{div}\, p^n \to \hat{u}$ as $n \to \infty$, where $\hat{u}$ is the solution of problem (1) and $\hat{v} = f - \hat{u}$.

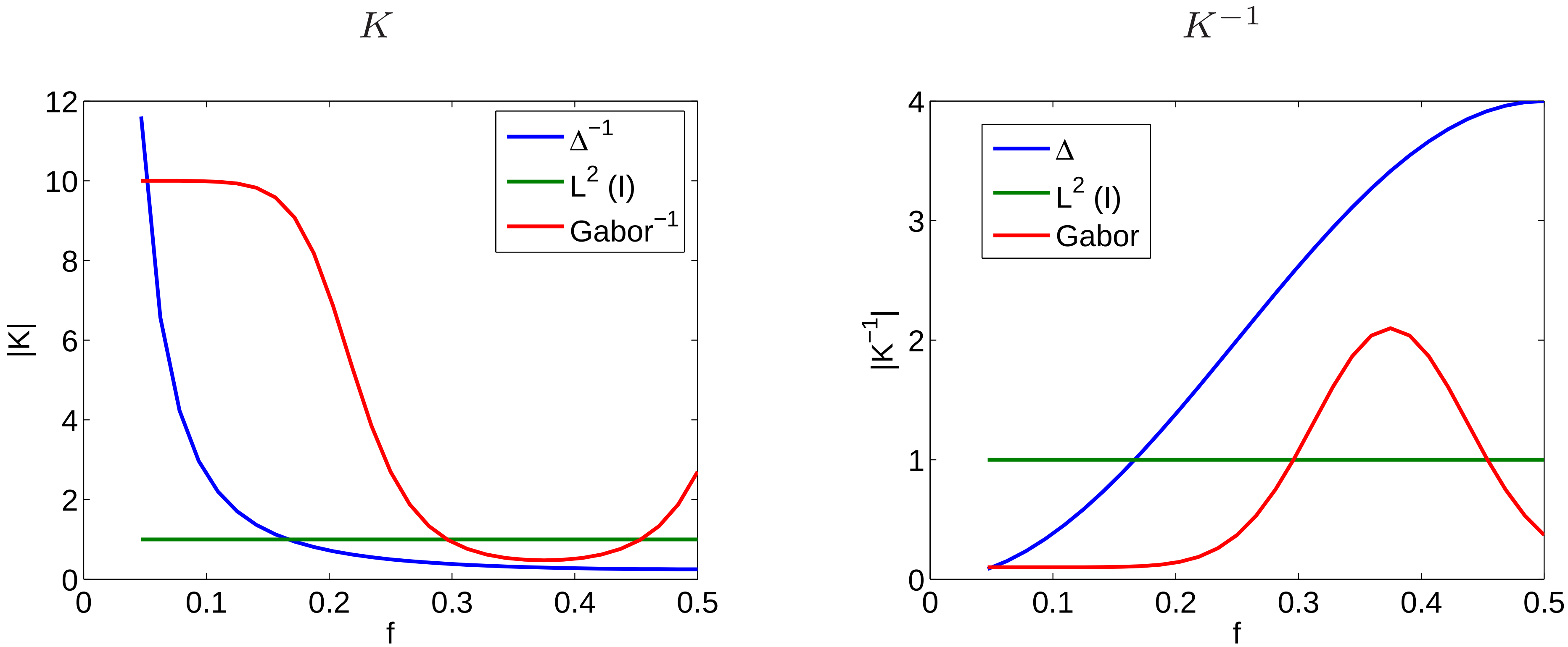


FIG. 1 – The kernel K and its inverse K^{-1} for the OSV, ROF and the proposed TV-Gabor model.

Texture

Textures are periodic elements.

$\Longrightarrow$ A simple way to characterize a texture is by its main frequency and direction.

$\Longrightarrow$ This naturally leads us to consider Gabor functions :

1D Gabor function (frequency ν, bandwidth σ) :

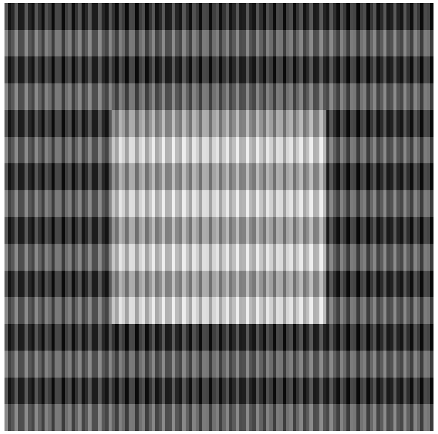
$$g(x) = \cos(2\pi\nu x)\,\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-x^2}{2\sigma^2}\right)$$

2D : consider $g(x)g(y)$ or a rotationally symmetric Gabor function :

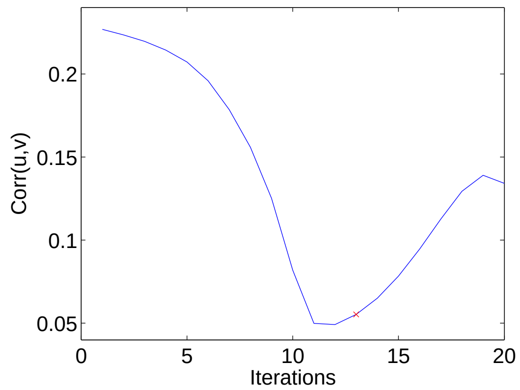
$$g(x,y) = \cos\left(2\pi\nu\sqrt{x^2+y^2}\right)\frac{1}{\sqrt{2\pi}\sigma^2}\exp\left(\frac{-x^2-y^2}{2\sigma^2}\right)$$

A simple example

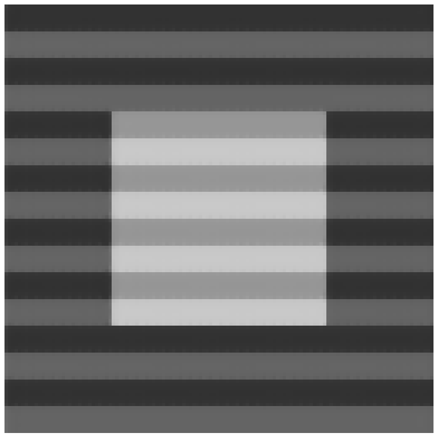
f



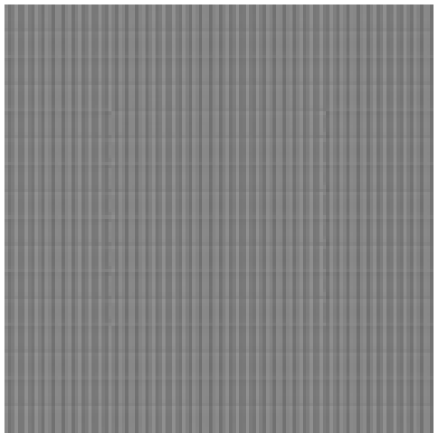
$\mathrm{corr}(u, v)$



u

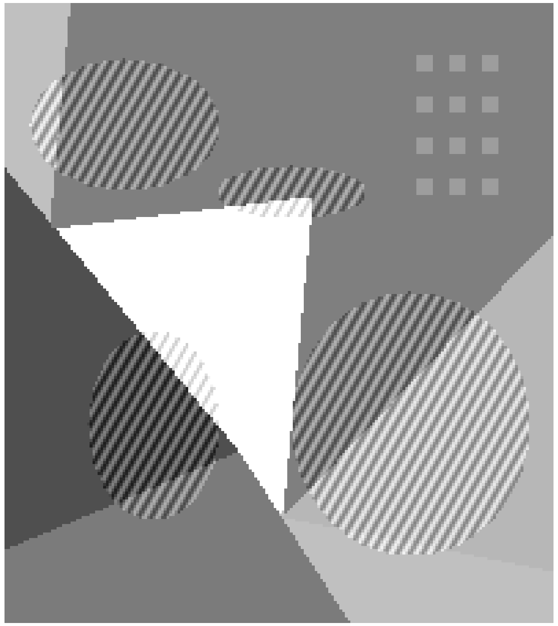


v



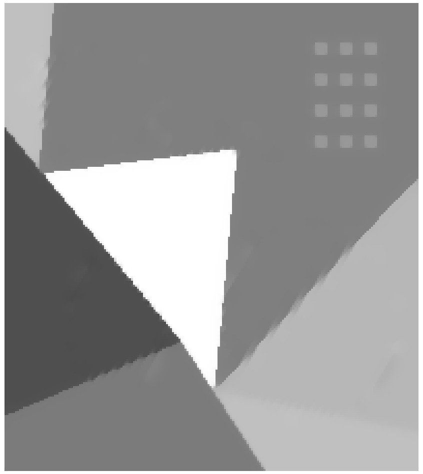
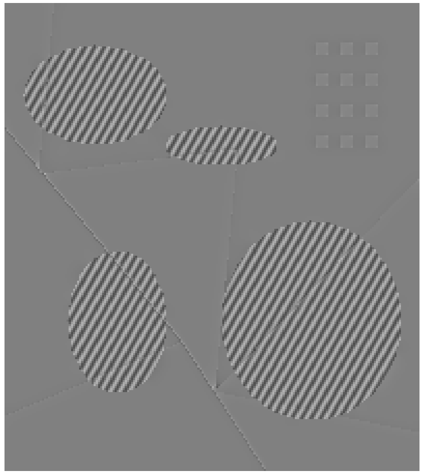
Results (I)

f

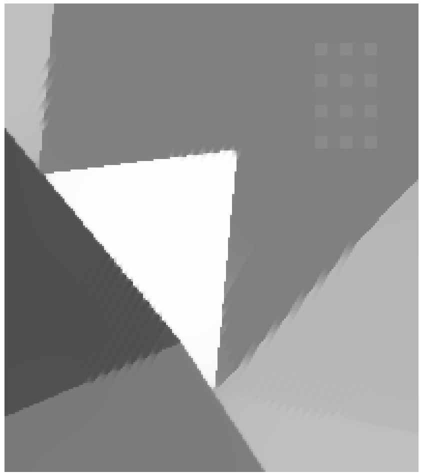
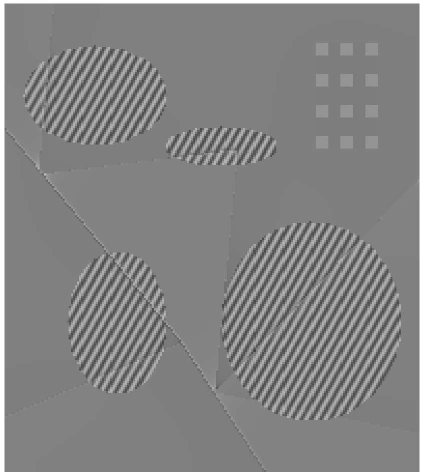


Results (II)

TV-Gabor, u $\qquad$ v

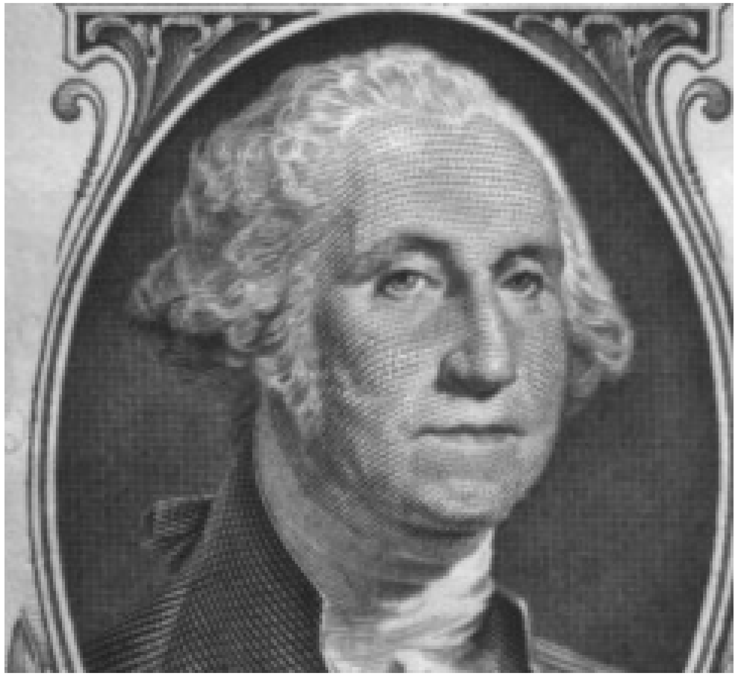
 

TV-L^2, u $\qquad$ v

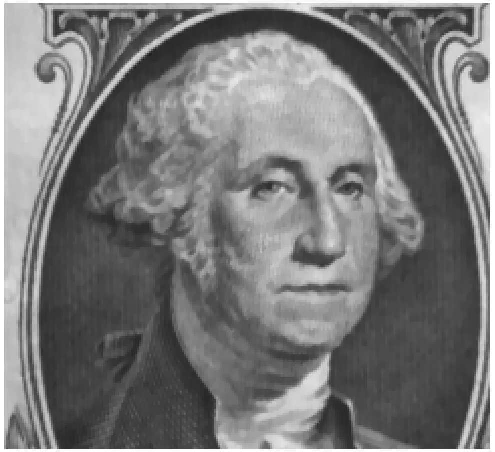
Washington (I)

f

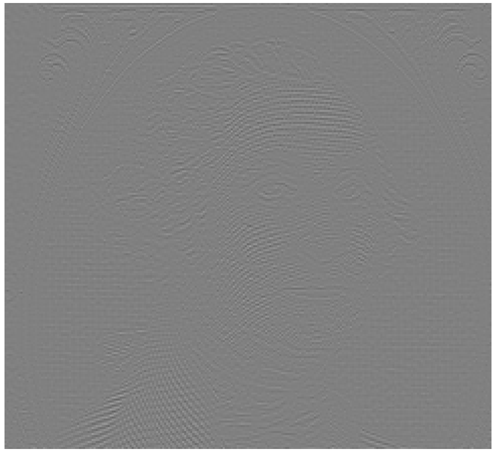


Washington (II)

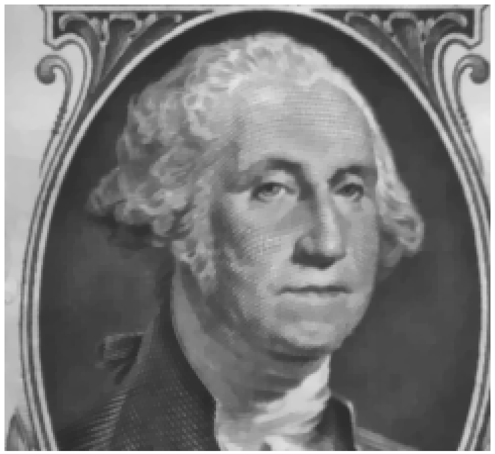
u (TV-Gabor)



v (TV-Gabor)



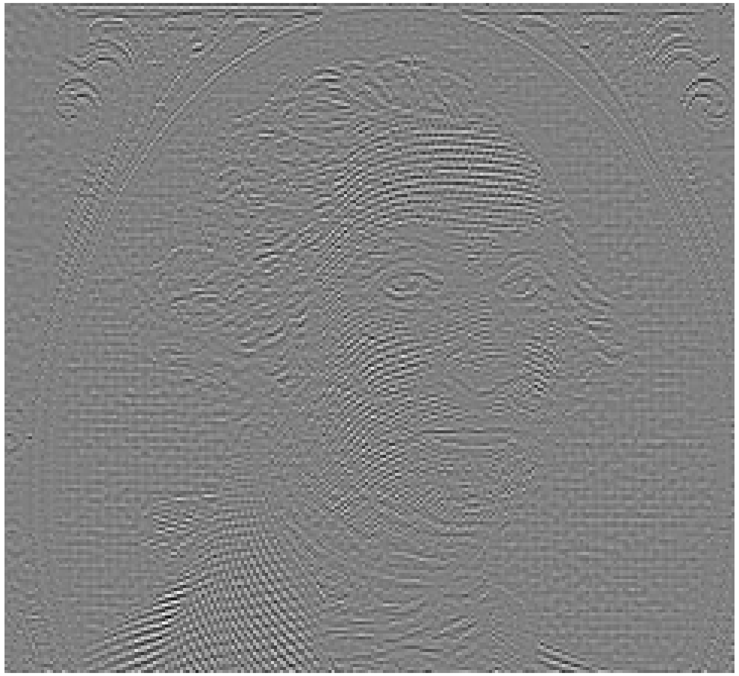
u $(TV - L^2)$



v $(TV - L^2)$

Washington (III)

4v (TV-Gabor)



4v ($TV - L^2$)

Overview

1) Problem statement

2) $TV + G$ model

3) $TV + G + E$ model

4) TV-Gabor model

5) Parameter selection

6) $TV - L^1$ model

Parameter selection problem

Aujol et al 2006

$$E_{Structure}(u) + \lambda E_{Texture}(v), \quad f = u + v,$$

$\Longrightarrow$ solution (u_λ, v_λ).

Problem : Find the right parameter λ.

$\Longrightarrow$ Very difficult problem (no σ^2 information as in the constrained denoising problem).

Simplest suggestion (based on Mrazek work on image denoising) :

Assumption : *The texture and the structure components of an image are not correlated.*

$$\lambda^* = \operatorname{argmin}_\lambda \left(\operatorname{corr}(u_\lambda, v_\lambda)\right).$$

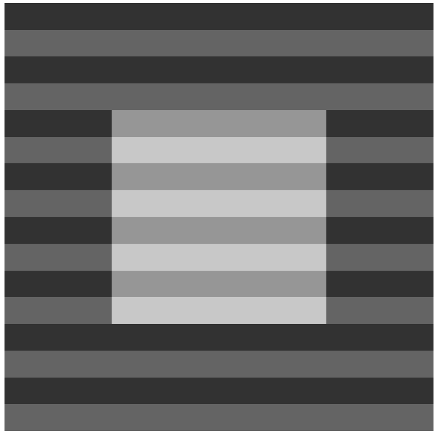
For the $TV - L^2$ model :

$$0 \leq \operatorname{corr}(u_\lambda, v_\lambda) \leq 1, \ \forall \lambda \geq 0.$$

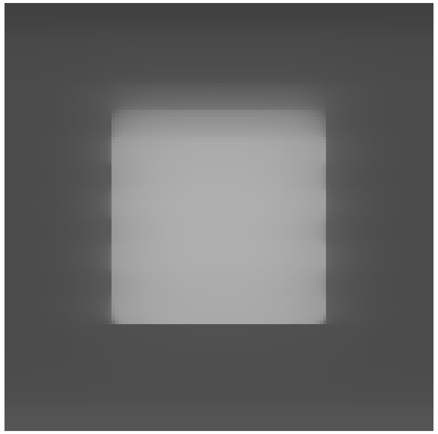
We do not claim that we pick the best possible parameter, but a good one.

A simple image

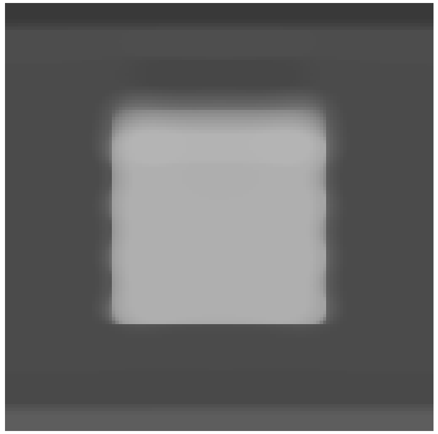
Original image



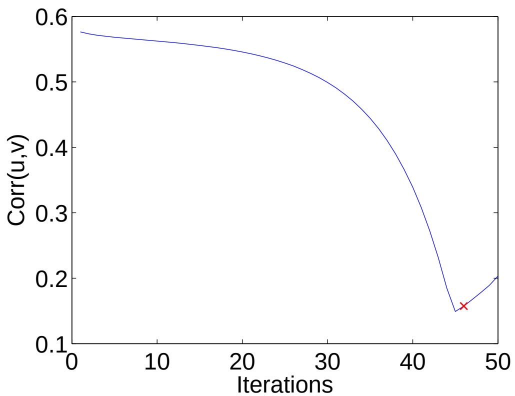
u_{ROF}



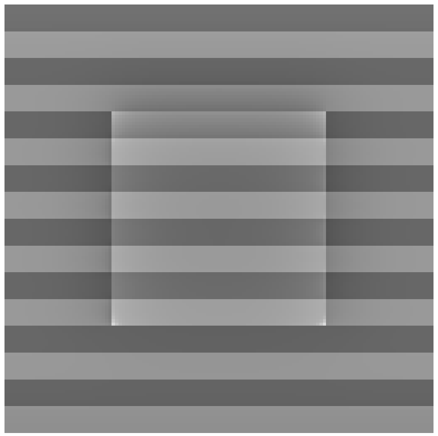
u_{A^2BC}



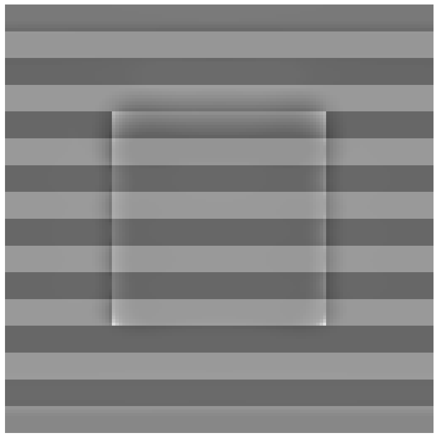
$\mathrm{corr}(u, v)$



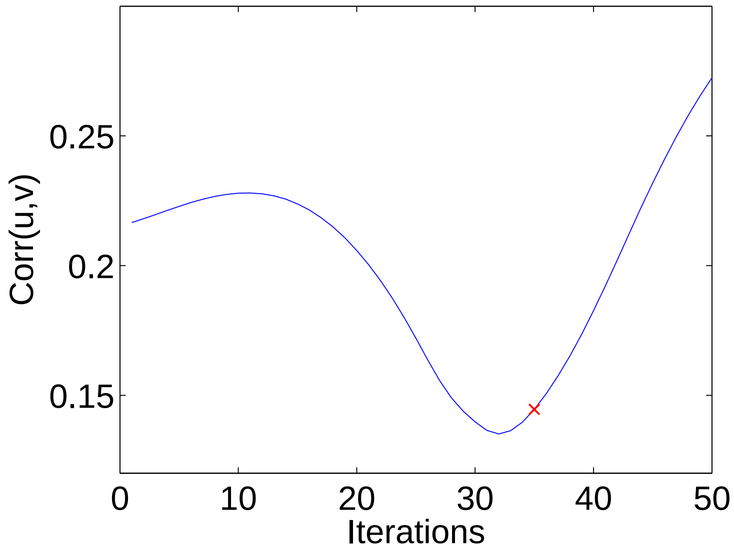
v_{ROF}



v_{A^2BC}



Barbara (I)



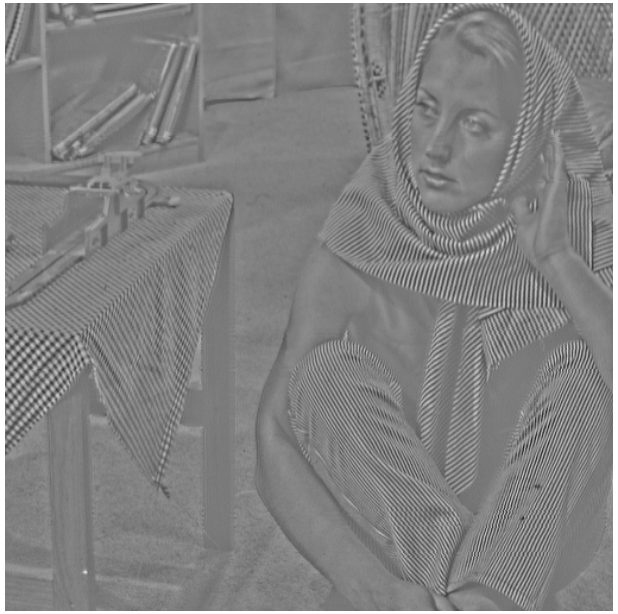
Barbara (II)

u_{ROF}

u_{A^2BC}

Barbara (III)

v_{ROF}



v_{A^2BC}

Overview

1) Problem statement

2) $TV + G$ model

3) $TV + G + E$ model

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5) Parameter selection

6) $TV - L^1$ model

Nikolova 2002 : relaxation algorithm

$$\inf_{(u,v)\in BV\times L^1 / f=u+v} \left\{ \int \sqrt{|\nabla u|^2 + \epsilon^2} + \lambda \|v\|_{L^1} \right\}$$

Chan-Esedoglu 2004 : PDE based algorithm

$$\inf_u \left\{ \int \sqrt{|\nabla u|^2 + \epsilon_1^2} + \lambda \int \sqrt{(f-u)^2 + \epsilon_2^2} \right\}$$

Aujol-Gilboa 2005 : projection algorithm

$$\inf_{u,v} \left\{ J(u) + \frac{1}{2\alpha} \|f-u-v\|_{L^2}^2 + \lambda \|v\|_{L^1} \right\}$$

Minimisation of the exact energy :

$$\inf_{(u,v)\in BV\times L^1 / f=u+v} \left\{ \int |\nabla u| + \lambda \|v\|_{L^1} \right\}$$

* Yinn et al 2006 (second order cone programming based algorithm)

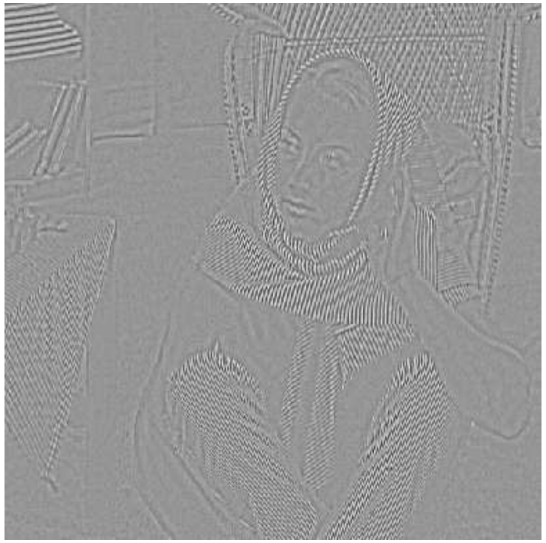
* Darbon-Sigelle 2006 (graph-cut algorithm, clearly the fastest algorithm to solve this problem)

Comparison

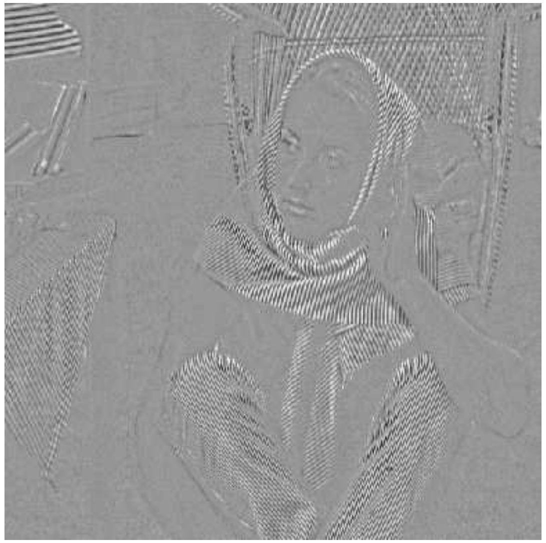
$u\ (TV - G)$

$u\ (TV - L^1)$

$v\ (TV - G)$



$v\ (TV - L^1)$



Conclusions

Image decompostion has been the subject of many studies during the last 6 past years. Just look at the UCLA CAM reports web page!

Luminita Vese (UCLA) and her students have proposed many other different functional analysis spaces to model textures (as originally suggested by Meyer in his book) : BMO, Besov spaces, negative Sobolev spaces, ...
It is not clear which one is the best choice.

Experiments indicate that L^1 might be a better choice ...(spatial scale more relevant than temporal scale ?)

J-L. Stark and his collaborators have studied the decomposition problem from a *sparse* point of view. Instead of looking for a norm which is small for textures, there are interested in a representation which is sparse for textures.

Future prospects

Future prospects :

Almost no work dealing with the problem of parameter selection.

No work dealing with the problem of spatial adaptivity.

Almost no work dealing with the problem of frequential adaptivity.

Developping the potential applications (inpainting, compression, classification)

Questions ?