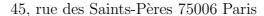
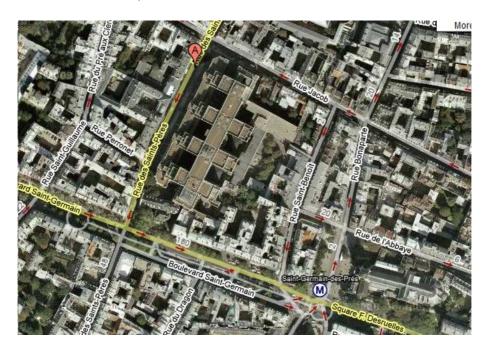
## Workshop on Models and Images for Porous Media

The aim of the workshop is to bring together researchers concerned with the studying, modelling, and imaging of porous media and their applications. Experts from different areas, from mathematics to physics and medicine, are invited. The topics will cover various aspects of porous media from mathematical models (random fields, morphology, fractals, etc) to practical applications (osteoporosis, mammograms, cements, etc). There will be tutorial lectures in the morning, invited sessions in the afternoon and a poster session will be organized. The workshop is organized as part of the project ANR-05-BLAN-0017 "mipomodim".

All the sessions will take place in the "Centre des Saints-Pères" building of the University Paris Descartes:





For the latest information, consult the website of the Workshop: http://mipomodim.math-info.univ-paris5.fr/

### Scientific Committee

Claude-Laurent Benhamou (CHR Orléans)

Christine Chappard (CHR Orléans)

Christine Graffigne (Université Paris Descartes)

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Anne Ricordeau (Université Paris 8)

workshop

#### Monday 12 Tuesday 13 Wednesday 14 Thursday 15 Friday 16 9:00 WelcomeR. J. Adler N. L. Fazzalari S. Jaffard 10:35 (Univ. Paris 12, France) (Technion, Haifa, Israel) (Adelaide, Australia) 11:00 J. Feder D. Jeulin M. Sahimi F. Peyrin W. Kendall 12:35 (ENSMP, Paris, France) (USC, USA) (CREATIS, Lyon, France) (Univ. of Oslo, Norway) (U. of Warwick, UK) Lunch and Lunch and Lunch and Lunch and End of the Poster Session 1 Poster Session 2 Poster Session 3 Poster Session 4 Workshop Mathematical Models Session 2: Lavoisier B Session 6: Lavoisier B Session 8: Lavoisier B Session 7: Lavoisier A & Spatial Statistics Session 5: Lavoisier A Fractal Parameters Random Geometry Microarchitecture Anisotropic Scale Session 3: Lavoisier Scaling Laws and Lavoisier Session 4: Lavoisier 14:30 Mathematical Porous Media Breast Image Transport in For Texture Morphology 16:00 Invariance Analysis Session16:30 18:00

Plenary lectures take place at Lavoisier A

• D. Jeulin

Monday 12, 11:00 - 12:30 (p. 10)

Mathematical Morphology, Random sets and Porous Media

• R. J. Adler

Tuesday 13, 9:00 - 10:30 (p. 12)

From statistics to topology and back again

• M. Sahimi

Tuesday 13, 11:00 - 12:30 (p. 13)

Efficient Multiscale-Scale Simulation of Transport and Reaction Processes in Heterogeneous Media: Application of Wavelet Transformations

• N. Fazzalari

Wednesday 14, 9:00 - 10:30 (p. 14)

Relationship Between Bone Morphometry (Trabecular And Cortical Bone) And Strength: Model And Experimental Data

• F. Peyrin

Wednesday 14, 11:00 - 12:30 (p. 15)

3D X-ray Micro-CT for the investigation of bone micro-architecture: image formation and analysis

• S. Jaffard

Thursday 15, 9:00 - 10:30 (p. 17)

Scale invariance in Signal and Image processing: Multifractal analysis and multifractal modeling

• J. Feder

Thursday 15, 11:00 - 12:30 (p. 16)

Fractal fronts in porous media flow

• W. S. Kendall

Friday 16, 11:00 - 12:00 (p. 18)

Networks and Poisson line patterns: fluctuation asymptotics, true geodesics and congestion

## **Invited Sessions**

## Session 1: Random Geometry and Spatial Statistics

Monday 12, 14:30 - 16:00 and 16:30 - 18:00

• J.-M. Azaïs 14:30 - 15:15 (p. 19)

Non asymptotic bounds for spatial statistics using the maxima of random processes

• W. Nagel 15:15 - 16:00 (p. 20)

A tessellation model for cell division or cracks

• V. Panaretos 16:30 - 17:15 (p. 21)

Modular Inference for Random Tomography in Structural Biology

• M. Lelarge 17:15 - 18:00 (p. 22)

Matchings on large random graphs

#### Session 2: Mathematical Models For Texture

Monday 12, 14:30 - 16:00 and 16:30 - 18:00

• G.-S. Xia 14:30 - 15:15 (p. 23)

Invariant morphological texture analysis

• J.-F. Aujol 15:15 - 16:00 (p. 24)

Image decomposition by variational methods

• P. Chainais 16:30 - 17:15 (p. 25)

Multifractal processes: from turbulence to images and textures

• G. Peyré 17:15 - 18:00 (p. 26)

Analysis and synthesis of turbulent dynamic textures

## Session 3: Transport in Porous Media

Tuesday 13, 14:30 - 16:00 and 16:30 - 18:00

• M.-O. Coppens

14:30 - 15:15 (p. 27)

Design of Hierarchically Structured Porous Catalysts with Minimal Diffusion Limitations

• J.-P. Korb 15:15 - 16:00 (p. 28)

Probing transport of liquids in porous media by NMR

• J. S. Andrade jr. 16:30 - 17:00 (p. 29)

Simulation of Non-Newtonian Fluid Flow in Porous Media

• G. Vignoles 17:00 - 17:30 (p. 30)

Heat and gas transport and chemical infiltration in fibrous media:  $\mu$ CT-based numerical methods

• R. Voituriez 17:30 - 18:00 (p. 31)

First-passage times and reaction kinetics in confined media

## Session 4: Anisotropic Scale Invariance: Models and Applications

Tuesday 13, 14:30 - 16:00 and 16:30 - 18:00

• C. Lacaux

14:30 - 15:00 (p. 32)

Operator scaling  $\alpha$ -stable random fields

• H.-P. Scheffler

15:00 - 15:30 (p. 33)

On simulation of operator-scaling stable random fields

• M. Clausel

15:30 - 16:00 (p. 34)

Explicit construction, classification of Operator Scaling Gaussian Random Fields. Some specific features of scaling exponent

• I. Tchiguirinskaia

16:30 - 17:00 (p. 35)

Generalized Scale invariance: the theoretical framework and its application to anisotropic turbulence and porous media

• Aline Bonami

17:00 - 17:30 (p. 37)

Anisotropy and Radon Transforms for Gaussian fields with stationarity properties

• J. Istas

17:30 - 18:00 (p. 38)

Anisotropy and Gaussian fields

#### Session 5: Bone Microarchitecture

Wednesday 14, 14:30 - 16:00 and 16:30 - 18:00

• C.-L. Benhamou

14:30 - 15:00 (p. 39)

Introduction to bone quality

• C. Chappard

15:00 - 15:30 (p. 40)

Correlation between 2D textural parameters and 3D bone micro-architecture morphology: generalities

• R. Jennane

15:30 - 16:00 (p. 41)

Correlation between 2D fractal parameters and 3D bone micro-architecture morphology

• A. Larrue

16:30 - 17:15 (p. 42)

Three-dimensional evaluation of micro-cracks in human trabecular bone

• S. Sevestre-Ghalila

17:15 - 18:00 (p. 43)

Cortical Bone Canal: Preliminary 3D Study

## Session 6: Mathematical Morphology

Wednesday 14, 14:30 - 16:00 and 16:30 - 18:00

• Viktor Benes 14:30 - 15:15 (p. 44)

Quantitative analysis of pores in a plasma-sprayed coating

• R. Hilfer 15:15 - 16:00 (p. 45)

Stochastic multiscale modeling of porous media

• A. Rack 16:30 - 17:00 (p. 46)

Analysis of spatial correlations in metal foams using synchrotron micro-tomography and 3D image analysis

• E. Le Trong 17:00 - 17:30 (p. 47)

Image analysis as a tool for the assessment of the weathering of building stones: preliminary results

• V. Tariel 17:30 - 18:00 (p. 48)

How to use the watershed transformation for the segmentation of granular materials and porous media obtained by X-ray tomography/SEM

## Session 7: Breast Image Analysis

Thursday 15, 14:30 - 16:00 and 16:30 - 18:00

• F. Clavel-Chapelon

14:30 - 15:15 (p. 49)

Breast density and mammogram aspect as risk factors for breast cancer

• C. Tromans 15:15 - 16:00 (p. 50)

Using a model of x-ray image formation to quantify breast tissue characteristics

• R. Zwiggelaar

16:30 - 17:15 (p. 51)

Mammographic risk assessment: linear structures and texture aspects

• P. R. Bakic 17:15 - 18:00 (p. 52)

Image-Based Biomarkers of Breast Cancer Risk: Analysis of Breast Density from Digital Breast Tomosynthesis Images

## Session 8: Scaling Laws and Fractal Parameters

Thursday 15, 14:30 - 16:00 and 16:30 - 18:00

• P. R. Bertrand 14:30 - 15:00 (p. 53)

Estimation of the Hurst index on finite frequency bands. Applications in health

• P. Abry 15:00 - 15:30 (p. 55)

Testing fractal connectivity in multivariate long memory processes

• Y. Xiao 15:30 - 16:00 (p. 56)

Anisotropic Random Fields and Their Fractal Dimensions

• A. Durand 16:30 - 17:00 (p. 57)

Large intersection properties and Poisson coverings

• Y. Achdou 17:00 - 17:30 (p. 58)

Some questions related to boundary value problems in some ramified domains with a fractal boundary

• D. S. Grebenkov 17:30 - 18:00 (p. 59)

Scaling Properties of the Spread Harmonic Measures

(p. 62)

#### Poster Session

• C. Redenbach, O. Wirjadi, H. Altendorf, M. Godehardt, K. Schladitz (p. 60) Characterization and Modelling of Microstructures Using Volume Images

• D. Bauer, S. Youssef, E. Rosenberg, S. Bekri, O. Vizika (p. 61)

Dual Pore Network Simulations Based on High Resolution  $\mu$ -CT Images to Calculate the Electrical Properties of Carbonates

• M. Couprie, J. Chaussard

Surface Patch Interactions with Topological Guarantees

• K. Hammoudi (p. 63)

Extracting 3D Polyhedral Building Models from Aerial Images

• R. Jennane, G. Aufort, C. L. Benhamou (p. 64)

Hybrid Skeleton Graph Analysis of Disordered Porous Media. Application to Trabecular Bone

• E. Koenig (p. 65)

Spherical Multifractal Texture Synthesis: Computer Graphics, Astrophysics and Analysis

• T. Lemaire, J. Kaiser, V. Sansalone, S. Naïli (p. 66)

A Multiscale Modelling of Fluid Transport within Bones: Consequences of Electrokinetics in the Mechanotransduction of Bones Remodelling Signals

• A. Ayache and Q. Peng (p. 67)

Stochastic Volatility and Multifractional Brownian Motion

• M. C. Stroe, M. Predoi-Racila, J. M. Crolet (p. 68)

Mathematical Modelization for Computing the Permeability in Human Cortical Bone

• C. Thäle (p. 69)

Self-Similar Fractal Mosaics

• S. Mase (p. 70)

On a Simulation of Gaussian Random Fields

• A. Brouste (p. 71)

Fractional Stochastic Processes and Fields in Geology

• D. Bernard, M. Giton, P. Benezeth (p. 72)

A Pore Scale Modelling of the Dissolution of the Calcite in a Reactive Flow of Water Saturated by CO<sub>2</sub> Flowing through a Carbonated Sample

## Mathematical Morphology, Random sets and Porous Media

by Dominique Jeulin

Centre de Morphologie Mathématique, Mathématiques et Systèmes, Mines ParisTech 35, rue St-Honoré, F77300 Fontainebleau, France Centre des Matériaux P.M. Fourt, Mines de ParisTech B.P. 87, 91003 Evry Cedex

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Since the seminal work of G. Matheron on porous media [11], and thanks to its implementation by means of image analysis [15], many applications based on mathematical morphology were developed. In this lecture, we will address the following points:

- The initial purpose of mathematical morphology was to extract quantitative information on microstructures. The first part of the lecture will introduce basic operations and measurements of mathematical morphology and their impact for studying porous or multicomponent media: in addition to stereological measurements, it is possible to estimate size distributions, scales, anisotropy and connectivity of a porous network. Recent advances in 3D image acquisition (usually by means of X-ray microtomography, and now by means of electron nanotomography) made possible direct 3D measurements, and to get information on the connectivity like the morphological tortuosity deduced from geodesic propagations [2].
- A complementary approach of microstructures is based on the theory of random sets [11, 13], giving a strong theoretical framework for modeling heterogeneous microstructures by means of a probabilistic approach. Models of random sets provide representations of microstructures and ways for simulations, in order to predict morphological as well as physical properties of complex media. This purpose will be illustrated by some basic models (using a combination of a Poisson point process and of random grains) and their applications to real data: the Boolean model [11, 13, 15, 16, 5], the dead leaves model [4, 5], and multiscale versions obtained from a Cox point process. They can show very different percolation thresholds [7, 8], which control their transport properties. Other models, derived from solutions of reaction-diffusion equations, provide natural textures [3].
- The theory of random sets also provides tools to solve homogenization problems, in order to predict the macroscopic properties of random media [11, 1, 10, 6]. The second part of the book of G. Matheron [11] is devoted to the hydrodynamics of porous media, namely the emergence of Darcy's law and to the composition of permeability. Few "exact" results are available, but it is possible to give some bounds of overall properties (permeability, coefficient of diffusion, conductivity, elastic moduli,...). The availability of 3D images (or the generation of simulations of realistic models of microstructures) combined to solvers of PDE gives access to the direct estimation of the effective properties: the coefficient of diffusion of a porous medium can be estimated from random walks [2]; the elastic moduli and the thermal conductivity can be obtained by finite elements [9]; using iterations of FFT [14], it is possible to solve the equations of elasticity on images without mesh; this was applied to the estimation of the elastic bulk modulus of a porous Boolean model [17]. For all these numerical estimations on 1 finite domains must be solved the question of the RVE (Representative Volume Element), which we address by a statistical approach.

#### References:

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- [14] Moulinec H., P. Suquet (1994): "A fast numerical method for computing the linear and nonlinear mechanical properties of composites", C.R. Acad. Sci. Paris, 318, Serie II, pp. 1417-1423.
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- [17] Willot F., Jeulin D. (2008) Elastic behavior of materials containing boolean random sets of inhomogeneities, International Journal of Engineering Sciences, in press.

### From statistics to topology and back again

by Robert J. Adler

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My plan is to start with a "simple" statistical problem that arises in a number of areas of statistical image analysis, including problems of mapping the brain and of analysing cosmological data from the early days of the universe. I will then show that the solutions to these problems lie in the study of smooth random processes parameterised by stratified manifolds.

The next step will be to look at the "excursion sets" of these processes, these being the (random) submanifolds of the parameter space over which the process exceeds given values.

We shall then see how recent results enable the development of explicit formulae for the mean values of a variety of geometrical characteristics of excursion sets for a wide variety of random processes, both Gaussian and non-Gaussian, and how these formulae relate to exceedence probabilities for random fields.

The first part of the lecture will concentrate on describing the main results and the brain mapping and cosmological applications. The second part will concentrate on extensions of the main results and the basic ideas behind the proofs. These involve some rather unexpected tools, including the kinematic fundamental formulae of Integral Geometry, Poincare's limit theorem for uniform variables on the n-sphere, and versions of the Weyl tube formulae.

The new results that will be discussed are the result of joint work with Akimichi Takemura and Jonathan Taylor.

## Efficient Multiscale-Scale Simulation of Transport and Reaction Processes in Heterogeneous Media: Application of Wavelet Transformations

by Muhammad Sahimi

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Transport processes in heterogeneous media are of fundamental importance to understanding a variety of phenomena in many systems of practical interests. Examples include transport in large-scale porous media (oil reservoirs, groundwater aquifers), ac conduction in amorphous seminconductors at low temperatures, and turbulent transport and reaction in the atmosphere. Two important characteristics of such system are, (1) their multiscale structure in which the disparity between the smallest and largest length scales of important emay be several orders of magnitude, and (2) their heterogeneity manifested by, for example, a broad spatial distribution of a local property, such as a transport coefficient or another important feature. Simulation of transport processes in such media is highly difficult, and almost always entails using massively-parallel computation methodology or supercomputers. We describe a novel computational algorithm, based on the use of the wavelet transformations, for generation of the computational grids that are highly resolved over the scales at which important information must be preserved in the grid, but coarsened where a fine grid structure is not needed. The grid may be static, or vary dynamically. The efficiency and accuracy of the method is demonstrated by its applications to three distinct classes problems, namely, low-temperature ac conduction in heterogeneous semiconductors, atmospheric pollution, and multiphase flow in a large-scale porous medium. In all the cases the method results in at least three orders of magnitude savings in computation time. The application of the method to several other important problems is also briefly described.

## Relationship Between Bone Morphometry (Trabecular And Cortical Bone) And Strength: Model And Experimental Data

by Nick L. Fazzalari

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The mechanical function of the skeleton is to sustain the loads applied by physiological motion. This skeletal optimisation relates to both the material properties of bone and their structural geometry. If a bone is too small or its structure degraded, if the bone material properties have been degraded, it is prone to failure under physiological load. This mechanical perspective of bone strength has prompted a strong interest in the concept of bone quality. This term relates to characteristics of bone, such as: trabecular and cortical bone architecture, the rate and extent of bone turnover, the organic and inorganic composition of bone matrix, the type and amount of collagen cross links, the degree of matrix mineralization, microdamage accumulation, and cell viability. These characteristics of bone quality influence the mechanical effect of complex physiological loads on the skeleton. Key areas to be discussed include bone remodeling, trabecular bone structure emerging from the growth plate and microdamage in bone. Bone remodelling redistributes and renews bone material between trabecular and cortical bone compartments. These compartments can have a significant effect on the mechanical behaviour of the skeleton, even when bone volume fraction remains unchanged. Non-uniform trabecular loss and adaptive changes in cortical bone may compensate for, or accentuate, the mechanical effects of trabecular loss. Growth plate morphometry, from the level of the chondrocyte resting zone through to the mature secondary trabecular bone structure is little understood. The trabecular bone architecture that evolves and is in place at maturity may form the template for all subsequent trabecular remodelling activity and pathologic architectural transformations. The process by which microcracks initiate, propagate, and ultimately coalesce leading to bone failure remains vague. Confocal images have identified evidence of the involvement of tissue structures at the submicroscopic level in fatigue damage, in addition to the potential influence of vascular canals and osteocyte lacunae on crack propagation through the bone matrix. The urgent need to evaluate the basis of how structural and material characteristics of bone relate to bone strength and fracture requires an integrated understanding of these tissue level factors.

## 3D X-ray Micro-CT for the investigation of bone micro-architecture: image formation and analysis

by Françoise Peyrin

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X-ray micro tomography (micro-CT) combines the principle of X-ray tomography, a popular technique in medical imaging and microscopic imaging at micrometric scale. This technique which allows the non destructive and three-dimensional investigation of porous media has received a considerable interest in the past decade. This growth is related to dissemination of commercial micro-CT systems thanks to technological advances in detectors and computers.

This talk will be divided in two parts, one devoted to the principle of micro-CT and the second related to 3D image analysis methods particularly used in the field of bone micro-architecture.

#### 1. Image formation in micro-CT:

We shall first review the principles of X-ray tomographic imaging, based on data acquisition and image reconstruction. We shall describe the different types of acquisition geometry in 2D and 3D. Basic 2D tomographic reconstruction algorithms based on the Radon Transform will be presented. The extension of these methods to 3D image reconstruction will be described in the case of parallel or cone-beam geometry. Finally we shall present the principle of synchrotron micro-CT, and its advantages over standard micro-CT for quantitative imaging at high spatial resolutions (up to the micrometer level).

#### 2. 3D Image analysis:

X-ray Micro-CT is now able to provide large 3D images of various porous media in short acquisition time. The exploitation of these data requires adapted image analysis methods. We shall particularly describe methods used for the quantification of bone micro-architecture. Image segmentation, which is the first step in image analysis, is fundamental since it conditions all the subsequent results. The next step is the extraction of quantitative parameters regarding the spatial organization of the medium. Morphologic parameters may be computed using stereology-based methods or model independent method. The great advantages of the later are that they do not rely on any assumption regarding the materials. Topologic parameters really benefit of the 3D nature of images which allows computing unbiased parameters. Global parameters based on Betti numbers as well as local parameters based on skeletons may be calculated. The different methods will be illustrated by applications to trabecular and cortical bone images acquired at 10  $\mu$ m resolution using synchrotron micro-CT.

## Fractal fronts in porous media flow

by Jens Feder

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In the first part, we consider fractal fronts in porous media flow. The displacement of one fluid by another fluid in homogeneous porous media leads to fractal fronts that depend on the flow rate. Also the dispersion of tracers in porous media exhibits an interesting fractal structure within the width of the front described by the continuum convection dispersion equation.

In the second part, we consider flow of particles in a microscopic pore, a 1-dimensional porous medium. Here we study the electrical, or optical, signals of particles that pass through a pore. The Hurst analysis shows that voltage fluctuations exhibit a very slow crossover from persistent to Brownian noise. We also measured the dissolution of microscopic droplets of hexane in water.

## Scale invariance in Signal and Image processing: Multifractal analysis and multifractal modeling

by Stéphane Jaffard

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Multifractal analysis supplies collections of parameters which allow to classify signals and images. Efficients numerical methods are now obtained by founding this analysis on wavelet leaders, which are defined as local suprema of wavelet coefficients. Nonparametric statistical tests are attached to the determination of these parameters in order to obtain sharp error bound, a mandatory requirement for applications in classification and model selection. This type of analysis can be applied to any type of function whithout a priori assumptions on its nature.

On the other hand, multifractal models display remarkable smoothness properties: The sets where their pointwise regularity exponent takes a given value are fractals, with varying Hausdorff dimensions. Two large classes of multifractal models have been considered up to now: additive models (e.g. random wavelet series), and multiplicative models (cascades).

The first part of the talk will give an overview of the methods used in multifractal analysis and modelling, with several applications in signal and image processing. The second part will focus on specific problems that appear in image processing, and on possible extensions to the analysis of sets with fractal boundaries and porous media.

This talk describes joint work with P. Abry, S. Roux, B. Vedel and H. Wendt (ENS de Lyon)

# Networks and Poisson line patterns : fluctuation asymptotics, true geodesics and congestion

by Wilfrid S. Kendall

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Poisson line patterns can be used to augment general large-scale networks so as to provide short-length routes at low cost [1]. I will survey this work, which uses the probabilistic method and expectation asymptotics to show how randomized constructions deliver the required augmentation. I will then discuss more recent work: control of random fluctuations in short routes in Poisson line patterns, typical geodesics, and congestion issues. This is joint work with D. J. Aldous.

#### Reference

[1] Aldous and Kendall, Advances in Applied Probability, 40:1, 1-12, 2008.

## Non asymptotic bounds for spatial statistics using the maxima of random processes

by Jean-Marc Azaïs

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For spatial statistics an alternative to the use of asymptotic expansion, that are true only for large level, is the use of bounds that ensure the control of the level of the test.

We present two methods. The first one considers two dimensional random fields, the most frequent case. It is based on the record method. The other, which applies to any dimension of the parameter, uses results from random matrices theory and in particular the Gaussian Orthogonal Ensemble GOE.

### A tessellation model for cell division or cracks

by Werner Nagel

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Several geometric structures can be modeled as tessellations that arise from a consecutive division of 'cells'. This concerns not only biological cells but also crack systems in materials science or in geology. A variety of stochastic models is already introduced and studied in literature, e.g. by Noble, Gray et al., Miles, Cowan, Kieu. In 2005 the STIT model (i.e. stable under iteration/nesting of tessellations) was introduced by Nagel and Weiss. These tessellations can be treated theoretically much better than other ones, and a series of results and formulas is already at hand.

In the lecture the STIT tessellations are explained and some properties and quantities are summarized. Several examples of potential applications are presented. As criteria for the goodness-of-fit the contact distribution functions as well as the K-function for the cell boundaries are considered.

This is joint work with Joseph Mecke, Joachim Ohser and Viola Weiss

## Modular Inference for Random Tomography in Structural Biology

by Victor Panaretos

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What can be said about an unknown density function on  $\mathbb{R}^n$  given a finite collection of projections onto random and unknown hyperplanes? This question arises in single particle electron microscopy, a powerful method that biophysicists employ to learn about the structure of biological macromolecules. The method images unconstrained particles, as opposed to particles fixed on a lattice (crystallography) and poses a variety of problems. We formulate and study statistically one such problem, namely the estimation of a structural model for a biological particle given random projections of its Coulomb potential density, observed through the electron microscope. Although unidentifiable (ill-posed), this problem can be seen to be amenable to a consistent modular statistical solution once it is viewed through the prism of shape theory.

## Matchings on large random graphs

by Marc Lelarge

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We study matchings on sequences of diluted random graphs converging to trees (for the local weak convergence). We prove the 'cavity' prediction for the limiting free energy density and show its relation with the resolvent of the limiting tree seen as an operator. As a result, we compute the limit for the rank of the sequence of graphs (joint work with C. Bordenave).

## Invariant morphological texture analysis

by Gui-Song Xia

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We present a new texture analysis scheme which is invariant to various local geometric and radiometric changes. The proposed methodology relies on the topographic map of images, obtained from the connected components of level sets. This morphological tool, providing a multi-resolution and contrast-invariant representation of images, is shown to be well suited to texture analysis. We first make use of invariant moments to extract geometrical information from the topographic map, involving first and second order statistics on shapes. This yields features that are invariant to local similarity or affine transforms. These features are invariant to any local contrast change. We then soften this invariance by computing additional features that are invariant to locally affine contrast changes and illustrate the resulting analysis scheme by performing classification and retrieval experiments on three texture databases.

## Image decomposition by variational methods

by Jean-François Aujol

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In this talk, I will review several variational models proposed for image decomposition during the last six past years. The problem of image decomposition is the following: given an original image f, one wants to split it into two components u+v. The first component u should contain the geometrical information of the image, it can be viewed as a sketch of f. The second component v should contain the oscillating patterns of the original image f, and in the case when f is noise free, v should be the texture component.

Inspired by a book by Y. Meyer (2002), a large body of literature has been devoted to the analysis of models and algorithms to compute such a decomposition. The idea is to minimize some energy of the type G(u)+T(v). G is a norm adapted to the geometry, and T a norm adapted to the texture (in the sens that T(v) is small if v contains only textures). G(u) is often chosen as the total variation of u, T(v) is often chosen as some negative Sobolev norm. The question of the choice of T will be discussed in this talk.

## Multifractal processes: from turbulence to images and textures

by Pierre Chainais

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For a long time, many tools have been developed by physicist to analyse data measured in turbulent fluid flows. However, the paradox is that many ideas and concepts developed to describe the experimental data could not be reproduced in any synthetic model to test, validate or illustrate the analysis tools. During the last ten years, various works have contributed to provide a variety of stochastic processes with nice properties such as scale invariance, multifractal behaviour, intermittency... While the first constructions were in 1 dimension only, they have been generalized to N dimensions, which gives rise to the possibility of image (2D) or porous media (3D) modeling for instance. Moreover, these processes can be numerically synthesized which results very useful for applications. We will present the family of multifractal processes viewed by a physicist and recall the initial motivations of physicist to construct multifractal analysis and scale invariant processes. We will present a large family of multifractal processes and focus on those built on infinitely divisible cascades in particular. Then we will present an application to the modeling of images of the Sun taken by the spatial telescope EIT and evoke some perspectives for texture synthesis.

## Analysis and synthesis of turbulent dynamic textures

by Gabriel Peyré

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In this talk I will present a new grouplet transform to process locally parallel textures. This transform is well suited to the modeling of natural textures. Unlike previous transforms routinely used for texture analysis, grouplets provide a sparse representation of directional textures. Indeed, transforms like wavelets or Gabor fail to compress geometric patterns present in natural textures. The grouplet transform is implemented with a fast adaptive algorithm that extracts a geometric flow and filters recursively the texture along this flow. The resulting transformed coefficients correspond to the decomposition of the image on a multiscale tight frame adapted to the image content. The grouplet coefficients of a geometric texture are both sparse and nearly independent, which makes this representation suitable for various texture processing tasks. I will show applications to texture inpainting and dynamic texture synthesis, which both require the joint optimization of the geometric flow and the grouplet coefficients.

## Design of Hierarchically Structured Porous Catalysts with Minimal Diffusion Limitations

by Marc-Olivier Coppens

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In biology, an appropriate structure of the distribution channel network from molecular to macroscopic scales is required for the proper functioning of, e.g., lungs, trees and cell membranes. In our work, this insight is used to guide the multi-scale design of novel catalysts, membranes, and reactors. Given our increased ability to synthesize hierarchically structured porous media with a desired structure at all length-scales, the practical implementation of theoretical designs is within reach.

Recent years have witnessed a revolution in our abilities to impose structure at the nanoscale. Properties like nanopore size and atomistic structure of the pore surface directly influence the intrinsic function of nanoporous materials, e.g., the intrinsic activity and selectivity in heterogeneous catalysis. To obtain a large specific surface area, these materials enclose an enormous array of interconnected nanopores. Because diffusion through these pores is slow, the benefits of nanoscale control are lost if the larger scales are neglected. Larger pore channels are needed to transport molecules toward and away from the catalytic sites, fast enough to counter diffusion limitations.

The questions are what the size distribution of these pores should be, and how the pores should be spatially distributed? This presentation will address the optimization of the multiscale pore structure of porous materials, from pore surface roughness to the largest pore channels traversing a particle, in order to meet a desired target, such as maximum yield or selectivity in a chemical process. Diffusion through the pores is studied using methods from statistical physics. The multi-scale optimization is based either on explicit pore network models or on continuum models, in which case multi-grid methods are particularly powerful.

The optimized structures differ significantly from presently used designs, and are closer to physiological structures. A properly designed channel network for molecular transport can drastically increase efficiency in catalysis, while preserving the intrinsic, microscopic activity and selectivity as much as possible. For the purpose of scale-up and homogeneous distribution, it often appears favorable to use fractal structures to interpolate between macroscopic and mesoscopic length scales. On mesoscopic length scales, a uniform distribution of pore channels leads to the highest yields if transport is limited by molecular diffusion, although other structures might perform better for different constraints. A few practical examples will demonstrate that significantly more might be achieved with much less material, in a way that is practically realizable. For example, twice the amount of noxious nitrogen oxides can be removed from exhaust gases, using half the amount of catalyst at typical commercial operating conditions.

## Probing transport of liquids in porous media by NMR

by Jean-Pierre Korb

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Information on liquid dynamics at solid surfaces is central to understanding transport properties in heterogeneous porous systems such as silica glasses, catalytic materials, rocks and cement-based and plaster materials. Nuclear magnetic relaxation dispersion (NMRD), the measurement of nuclear spin-lattice relaxation rates (1/T1) as a function of magnetic field strength or nuclear Larmor frequency, provides a powerful approach for characterizing molecular dynamics, including surface dynamics. This approach gives information about the structure and dynamics of a liquid layer near a solid boundary surface. This is all the more important as a liquid confined to a region that approaches molecular dimensions has dynamic and thermodynamic properties strongly changed relatively to the bulk. Examples include anomalous diffusion, chemical exchange and anomalous temperature behaviour, shifts of melting and freezing temperatures, anisotropic molecular reorientation at pore surface.

We will detail our recent NMRD experiments to provide reliable information on the progressive setting of the microstructure of cement-based materials. The remarkable features of the relaxation dispersion support an interpretation in terms of coupled solid-liquid relaxation at pore interfaces, surface diffusion and nuclear paramagnetic relaxation. The measurement does not require any drying temperature modification and is sufficiently fast to be applied continuously during the progressive hydration of the material. Coupling this method with the standard proton nuclear spin relaxation and high resolution NMR allows us to follow the development of texture within the material. Another well-known example consists in the water dynamics at the growing solid interfaces of a plaster paste. Here, a new concept of nanowettability has been introduced that connects the fluid-solid interaction and the surface fluid dynamics.

Last, our recent applications of 2-dimensional T1-T2 and T2-store-T2 correlation experiments for tracking water exchange in connected micropores of various cement pastes will be also presented.

#### Simulation of Non-Newtonian Fluid Flow in Porous Media

by José S. Andrade jr.

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Fluid flow through porous media is common in many petroleum engineering applications, the design of chemical reactors, a variety of separation processes and polymer engineering. In this work, we investigate by numerical simulations the behavior of non-Newtonian viscous and non-viscous fluid flow through 3-dimensional disordered porous media. Our results for power-law fluids suggests that an universal curve can be representative of the macroscopic behavior of the flow system for low and moderate Reynolds numbers, regardless of differences in the porous medium porosity and in the power-law index (exponent) of the fluid. For the case of a Hershel-Bulkley fluid, our study reveals the existence of an "optimal" flow condition as a consequence of the complex interplay between viscous and inertial forces acting on the system.

## Heat and gas transport and chemical infiltration in fibrous media: $\mu$ CT-based numerical methods

by Gérard Vignoles

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We present simulation techniques aimed at the description of the transport and reaction of gases in continuum to rarefied regimes in large 3D blocks describing fibrous media with geometry evolution.

The application case is the preparation of Carbon/Carbon composites by Chemical Vapour Infiltration of a pyrocarbon matrix inside a complex arrangement of carbon fibres: hydrocarbons at low pressure diffuse inside the pore space and react with the solid phase to give a carbon deposit. Modelling this process at pore scale should address gas transfer, possibly in rarefied conditions, heterogeneous chemical reactions, and porous medium evolution. Moreover, thermal gradients may also be present, which motivates the development of heat conductivity evaluations.

High-resolution images of C/C composites are obtained by Synchrotron X-ray micro-CT in phase contrast mode, plus adapted image segmentation techniques. Modelling is addressed with the use of Monte-Carlo Random Walks for gas transport, Simplified Marching Cubes for the discretization of the fluid/solid interface, and a pseudo-VOF technique for the interface growth handling.

In lower-resolution images, heat transfer is computed by a volume averaging technique coupled with the detection of the local fibre orientations inside the image.

Results are presented and discussed with respect to experimental data and previous studies on ideal media.

## First-passage times and reaction kinetics in confined media

by Raphael Voituriez

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How long does it take a random walker to reach a given target point? This quantity, known as a first passage time (FPT), has led to a growing number of theoretical investigations over the last decade. The importance of FPTs originates from the crucial role played by first encounter properties in various real situations, including transport and reactivity in complex media, spreading of diseases or target search processes. I will propose here a general theory which allows one to evaluate the mean FPT (MFPT) in complex media. This analytical approach provides a universal scaling dependence of the MFPT on both the volume of the confining domain and the source-target distance. This analysis is applicable to representative models of transport in disordered media, fractals, and anomalous diffusion. I will last discuss applications to chemical reactions in confined media.

## Operator scaling $\alpha$ -stable random fields

## by Céline Lacaux

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The Fractional Brownian Motion, defined in [1], is a very powerful model in applied mathematics. However, it is isotropic and then it is not convenient for anisotropic media modeling. In order to describe the anisotropy of the media, [2] has introduced some anisotropic Gaussian and  $\alpha$ -stable random fields satisfying an operator scaling property. A scalar-valued random field  $(X(x))_{x \in \mathbb{R}^d}$  is called operator scaling for the matrix E if

$$\forall c > 0, \ \left(X\left(c^{E}x\right)\right)_{x \in \mathbb{R}^{d}} \stackrel{(f.d.d.)}{=} c\left(X(x)\right)_{x \in \mathbb{R}^{d}}$$

where  $c^E = \exp(\log(c)E)$ . Such random fields may satisfy some scaling relations that depend on the direction. Indeed, if the random field X is operator scaling for E and if u is an eigenvector of E associated with the eigenvalue  $\lambda \in \mathbb{R} \setminus \{0\}$  then

$$\forall \varepsilon > 0, \ (X(\varepsilon tu))_{t \in \mathbb{R}} \stackrel{(f.d.d.)}{=} \varepsilon^{1/\lambda} (X(tu))_{t \in \mathbb{R}}.$$

If X is a Gaussian operator scaling random field as defined in [2], then it has stationary increments and the critical pointwise and global Hölder exponents of its sample paths follow from the study of its variogram. These exponents may depend on the direction and are given by the real part of the eigenvalues of E.

In this talk, we will be interested in the sample paths properties of the  $\alpha$ -stable operator scaling random fields defined in [3]. In the case of harmonizable operator scaling stable random fields, the critical pointwise and global Hölder exponents follow from an upper bound of the modulus of continuity and are the same as in the Gaussian realm.

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## On simulation of operator-scaling stable random fields

by Hans-Peter Scheffler

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A large class of operator-scaling stable random fields (OSSRFs) was presented in [1]. In that paper, based on so-called E-homogeneous functions a moving average and a harmonizable representation were given. In this talk we present in dimension two, for a certain subclass of E-homogeneous functions, simulation algorithms for both moving average and harmonizable OSSRFs.

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## Explicit construction, classification of Operator Scaling Gaussian Random Fields. Some specific features of scaling exponent

by Marianne Clausel<sup>1</sup>, Béatrice Vedel<sup>2</sup>

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In [1] H. Biermé, M. Meerschaert, H. P. Scheffler constructed a large class of operator selfsimilar Gaussian or stable random fields with stationary increments. These fields satisfy the following scaling property

$$\left\{X\left(a^{E_0}x\right)\right\}_{x\in\mathbb{R}^d} = \left\{a^{H_0}X\left(x\right)\right\}_{x\in\mathbb{R}^d}.$$

for some matrix  $E_0$  (called an exponent of the field) whose eigenvalues have a positive real part, and some real  $H_0$  (called an Hurst index of the field). We focus on the Gaussian case and propose a classification of these fields from four generic cases. Moreover, we are able to propose an explicit construction of these fields and then extend the results of [1] in which spectral densities are defined by an (non-explicit) integral formula. Furthermore, we establish regularity results of the sample paths by measuring smoothness in general anisotropic Hölder spaces, not necessary related to the exponent matrix  $E_0$  of the studied field. We prove that from the regularity point of view, matrix  $E_0$  has some specific features. More precisely, in dimension 2, the critical exponent in anisotropic Hölder spaces linked to matrix  $E_0$  is maximal among all critical exponent in anisotropic Hölder spaces and equals Hurst index  $H_0$ . In greater dimensions, a weaker optimality result is obtained.

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## Generalized Scale invariance: the theoretical framework and its application to anisotropic turbulence and porous media

by Shaun Lovejoy<sup>1</sup>, Daniel Schertzer<sup>2</sup>, Ioulia Tchiguirinskaia<sup>3</sup>

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Scaling is the key to future developments in geophysics and more generally in nonlinear physics. Nevertheless, it requires a generalized notion of scale in order to deal both with scale symmetries and strong anisotropies, e.g. time vs. space, vertical vs. horizontal. Surprisingly, the Fractal Geometry [1] preserved the classical Euclidean distance to define the topology and the metric of the physical space, in particular to define the Hausdorff dimension. There are on the contrary at least three main reasons [2, 3-6] to question the relevance of this metric for most of the (geo-) physical processes. The first is that it is meaningless to assume isotropy between time and space because they are incommensurable. The second is that gravity induces in a general manner a strong anisotropy in space. Thirdly, there is no mathematical compelling reason to use the Euclidean distance to define the Hausdorff dimension and scales: a (generalized) notion of scales  $\|\cdot\|$  is readily obtained with the help of a one parameter  $(\lambda)$  anisotropic contraction semi-group  $T_{\lambda}$ :

$$||T_{\lambda}\underline{r}|| = \lambda^{-1}||\underline{r}||.$$

This notion turn out to be particularly indispensable to understand, analyse and model (geo-) physical phenomena ranging from atmospheric turbulence to porous media. With the help of a few examples [7, 8-15], we will show how GSI readily defines anisotropic phenomena or media; e.g. atmospheric motions and hydraulic conductivity are 23/9 D rather than being quasi-2D or quasi-3D. GSI can be understood as a radical paradigm shift with respect to the principle of local isotropy [16]: the basic principle becomes to posit first scaling, then to study the remaining symmetries. It is rather significant that this is no longer limited to fundamental and theoretical problems of geophysics, but now touches many applications including environmental management [17].

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# Anisotropy and Radon Transforms for Gaussian fields with stationarity properties

by <u>Aline Bonami</u><sup>1</sup>, José R. Léon<sup>2</sup>

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Different models, which are obtained as deformations of the Fractional Brownian Field  $B_H(t), t \in \mathbb{R}^2$ , have been introduced to model rough anisotropic images. A curious phenomenon of some of them is related to the fact that anisotropy is better observed and measured on the Radon transform of the field than directly, when looking at increments in different directions. We will in particular consider Gaussian fields X with stationary increments and spectral density f. For f given in polar coordinates by  $f(r,\theta) := r^{-\omega(\theta)}(1+o(1))$  at  $\infty$  for r, in a previous joint work with Ayache and Estrade we used Radon transform to give an estimator of  $\omega(\theta)$  for some fixed value of  $\theta$  (under some technical assumptions). In a recent work, Biermé and Richard have proved that these estimators are asymptotically independent when considering two different directions  $\theta_1$  and  $\theta_2$ , which allows them to propose a test of anisotropy. The same phenomenon of independence can be observed in other models, such as those where anisotropy is given in the constant, that is  $f(r,\theta) := \frac{\omega(\theta)}{r^{2H}}$ . This last case is considered in a joint work in progress with José Léon.

# Anisotropy and Gaussian fields

by Jacques Istas

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Self-similar fields with stationary increments may be anisotropical. We will recall their spectral representation and explain how can the anisotropy be estimated with shifted generalized quadratic variations.

### Introduction to bone quality

by Claude-Laurent Benhamou

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For many years, the evaluation of bone status concerning osteoporotic fracture risk was uniquely based on the Bone Densitometry Measurement (BMD). However as soon as 1993, the definition of osteoporosis proposed by the WHO Organization included both a decrease in bone mass (as reflected by BMD) and an alteration of trabecular bone microarchitecture [1]. This definition was revised for the WHO Organization by a group of experts in 2001 and included a decrease in bone mass associated to bone quality alterations [2]. The concept of bone quality is generally presented as a multiscale concept, from the organ by itself up to the molecular scale. At the microscopic scale quality bone properties include size, shape, cortical thickness of bones. The following scale includes microarchitecture of trabecular and cortical bone. At the microscopic scale, we can also determine the cellular behaviour and bone remodeling properties as well as the presence of microcracks, and the degree of bone mineralization. Bone strength at the microscopic level can be investigated by nano identation methods. Finally at the molecular scale, the changes involve collagen and bone crystal.

Among these bone quality components, the most largely studied is the trabecular bone microarchitecture. Its role is largely demonstrated and more and more work is developed to propose pertinent evaluations to the clinicians, in parallel to the BMD measurements.

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# Correlation between 2D textural parameters and 3D bone micro-architecture morphology: generalities

by Christine Chappard

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Dual X-ray Absorptiometry (DXA). However, 44% of patients with non vertebral fracture are not identified by this technique which does not capture all trabecular bone microarchitectural changes. Moreover, this technique is not largely used, but expensive and reimbursed only in

Osteoporosis is a bone disease that leads to an increased fracture risk. At this time, the definition of osteoporosis lies on the bone mineral density (BMD) measurements assessed by

Few techniques with sufficient resolution could provide in vivo access to 3D bone micro-architecture such as Quantitative Computed Tomography (QCT), peripheral QCT (pQCT), Magnetic resonance imaging (MRI). From these techniques, it is possible to assess usual morphological 3D trabecular bone parameters such as bone volume fraction (BV/TV), trabecular thickness (Tb.Th), separation (Tb.Sp) and trabecular number (Tb.N) as well as connectivity and anisotropy parameters. However, the main disadvantage of these techniques is to be limited in accessibility. On the contrary, conventional radiographs (2D projectional images) are widespread and low cost.

Most of textural methods are used on radiograph images and based on fractal or Fourier analysis, run length method, co-occurrence matrices (also known as Haralick's method) and textural method based on gradients. In clinical studies, few of these methods have been already tested successfully to discriminate patients presenting fragility fractures comparatively to controls.

The trabecular bone is a biphasic structure including marrow space and trabeculae which are organized in groups. Osteoporosis leads to change in the trabecular bone organisation. The alteration of trabecular bone is the result of a combination of Tb.N, Tb.Th and connections reductions paralleling a Tb.Sp increase. These variations induce variation of X-ray attenuation in the bone and consequently variation of transparency in the image and textural features. By mutivariate analysis, it is possible to analyse the correspondence between textural data and the real 3D bone micro-architectural structure.

The development in hospital of numerical radiology and picture archiving and communication system (PACS) which is a multiple users accessibility to imagery is in progress and probably soon largely available. These new developments could allow a large use of this textural analysis for the fracture risk assessment.

# Correlation between 2D fractal parameters and 3D bone micro-architecture morphology

by Rachid Jennane

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In this work, we propose a fractal analysis of bone X-ray Tomographic Microscopy projections (XTM). The aim of the study is to establish whether or not there is a correlation or a direct link between threedimensional (3-D) trabecular changes and two-dimensional (2-D) fractal descriptors. Trabecular bone loss was simulated using a mathematical morphology method. Then, 2-D projections were generated in each of the three orthogonal directions. Finally, the model of fractional Brownian motion (fBm) was used on bone XTM 2-D projections to characterize changes in bone structure that occur during disease, such a simulation of bone loss. Results indicate that fBm is a robust texture model allowing quantification of simulations of trabecular bone changes. Plus, results show that the self-similarity,  $S_{3D}$ , of the 3D bone volumes and that of their projections,  $S_{2D}$  are linked by  $S_{3D} = S_{2D} - 0.5$ . This demonstrates that a simple projection provides 3D information about the bone structure.

# Three-dimensional evaluation of micro-cracks in human trabecular bone

by <u>Aymeric Larrue</u><sup>1,2</sup>, Zsolt Peter<sup>2</sup>, Aline Rattner<sup>3</sup>, Laurence Vico<sup>3</sup>, Françoise Pevrin<sup>1,2</sup>

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While the understanding of micro-damages in bone tissue is raising increasing interest there are little data on micro-cracks in trabecular bone.

In this work, we used 3D Synchrotron radiation micro-CT (SR $\mu$ CT) at the micrometer scale to characterize micro-cracks in human trabecular bone.

Cylindrical samples (diameter 10mm) were extracted from human femoral heads taken out during arthroplasty.

While they were maintained in a physiological environment to preserve soft tissues, different cyclic fatigues (strain: 0.1%) were applied to 2 groups of samples ((1): 10Hz, 10 000 cycles; (2): 1Hz, 7200 cycles). A control group (3) was also studied. After that, 5mm x 5mm x 10mm parallelepipeds were cut and embedded in PMMA resin. For each sample, two regions of interest of (2.8mm x 2.8mm x 2.2m) were imaged with the SR $\mu$ CT system developed on beamline ID19 at the ESRF (Grenoble, France). The voxel size was 1.4 $\mu$ m and the energy was 24 keV.

Micro-cracks corresponding to the different types described in the literature (linear, parallel, cross-hatched) could be observed in the tomographic slices, showing that the spatial resolution was adapted. Moreover, we enumerated the number of cracks N.Cr per tissue volume, the number of cracks intercepting a lacuna (N.Cr $\cap$ La), following (N.Cr Bdr) or stopped (N.Cr $\perp$ Bdr) by the boundary between two areas of different mineralization.

We developed a semi-automatic process, robust to noise, low contrast and ring artefacts, to segment micro-cracks in 3D images. Since they are described as thin planar ellipses, we computed the best-fitting ellipsoids to access the cracks length (Cr.L) and width (Cr.w). We also estimated their thickness (Cr.Th) and volume (Cr.V). We present the average of the measures of all cracks in one sample of each group in Table 1. A large variability was observed within each sample but the results are in agreement with previously published works. This is the first 3D evaluation of micro-cracks in trabecular bone.

Sample in Group	1	2	3
BV/TV (%)	22,79	17,36	22,61
$N.Cr (\#/mm^3)$	0.66	0.80	0.53
Cr.V (mm <sup>3</sup> x106) mean±std	$19,70\pm25,98$	$9,52\pm25,89$	$10,04\pm12,12$
Cr.Th $(\mu m)$ mean $\pm$ std	$3,2\pm 1,1$	$3,5 \pm 1,5$	$3,2 \pm 1,1$
Cr.L $(\mu m)$ mean $\pm std$	$163,5\pm 89,2$	$92,9\pm122,5$	$114,7\pm78,2$
$Cr.w (\mu m) mean \pm std$	$87,7\pm62,1$	$38,1\pm41,8$	$58,6\pm43,3$
$N.Cr \cap La (\#/mm^3)$	0.47	0.07	0.33
$N.Cr.Bdr (\#/mm^3)$	0.53	0.40	0.07
$N.Cr.\perp Bdr (\#/mm^3)$	0.07	0	0

Table 1: Average parameters of all cracks in a sample of each group

### Cortical Bone Canal: Preliminary 3D Study

by S. Bensalah<sup>1</sup>, <u>S. Sevestre-Ghalila<sup>1,2</sup></u>, S. Bensamoun<sup>3</sup>, A. Basillais<sup>3</sup> and C. Chappard<sup>3</sup>

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3D trabecular bone micro-architecture has been largely investigated whereas very few studies exist concerning the cortical bone. The analysis can be symmetrical focalised on 3D canals instead of the trabecular bone structure. The porosity, the canal size, canal spacing, the number of canals are assessed based on usual methods developed for trabecular bone. However, the complexity of the canal structure can not be assed from these parameters. Indeed, the trabecularization of the cortical bone needs local evaluation of the canal regularity and connectivity changes. We have carried out this preliminary study on human femoral bone in order to better characterize local changing of cortical bone in terms of connectivity and regularity.

We used 11 samples extracted at lateral and medial sides from 3 femoral diaphysis imaged by a micro-CT at  $7.8\mu\text{m}^3$  (Skyscan 1072R). From sample of 600x600x500 voxels about 8000 connected canal components can be extracted after a pre-processing of thresholding and canal component labeling.

To optimize hard disk space and time computing, we ended the labeling of each component by saving it separately in a binary file. This modified labeling process divides the time computing of the features about 8.51 times and about 17.79 times with space saving. Thereby, we extract 4 canal features from 3D canal components: two regularity parameters and two connectivity parameters.

Principal Component Analysis applied on 200- 3D canal components randomly selected from each bone set of samples exhibits greater values of the regularity and connectivity features for lateral canals than medial ones. As results, we established the ability of the proposed canal features to distinguish between medial and lateral canals by using an optimal 3D canal component extraction with respect to time computing and space saving. As the features computation is based on rates of 2D connected part surface and gravity location through the canal component, we are currently working on including features computation in the labeling process in order to discriminate Havers and Volkmann.

# Quantitative analysis of pores in a plasma-sprayed coating

by Viktor Benes

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Quantitative microstructural analyses of thermally sprayed coatings are reviewed. Then a ceramic plasma-sprayed coating is analysed using light micriscopy and image analysis. Globular pores and interlamellar flat pores are reconstructed from serial sections of a specimen and their volume and surface area distributions are estimated, respectively. Spatial distribution of pores is described using a method based on 3D distances and testing of randomness is performed. Interpretations of results in terms of physical background of the material are discussed. This is joint work with P. Ctibor and R. Lechnerova.

### Stochastic multiscale modeling of porous media

by Rudolf Hilfer

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A method to generate stochastic morphologies for multiscale media is presented [1]. The method is particularly suited for modeling carbonate rocks occurring in petroleum reservoirs that exhibit porosity and grain structure covering several decades in length scales. The mathematical model reproduces correlations with primordial depositional textures, scale dependent intergranular porosity over many decades, vuggy porosity, a percolating pore space, a percolating matrix space, and strong resolution dependence of both physical and morphological descriptors such as permeability or Minkowski functionals. The continuum based model allows discretization at arbitrary resolution and provides synthetic micro-CT images for resolution dependent simulations, morphological analysis or tests of multiscale models and methods.

[1] B. Biswal et al., Phys.Rev.E, vol 75, 061303 (2007)

# Analysis of spatial correlations in metal foams using synchrotron micro-tomography and 3D image analysis

by <u>Alexander Rack</u>, L. Helfen, T. Baumbach, J. Banhart, K. Schladitz, J. Ohser

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We investigate spatial cross-correlations in metal foams to understand control mechanisms behind their pore formation. The metal foams are produced using the powdermetallurgical route: an alloy (e. g. AW-6061) to be foamed in powder form is mixed with a blowing agent (e.g. TiH2) and then compacted in order to create a solid pre-cursor material. The pre-cursor is heated in a furnace. In an ideal case at the same point where the alloy transforms from solid into a mushy state the blowing agent starts to release gas which forms the pores. Quenching of the sample at the desired expansion state conserves the pore structure, resulting in a metal foam with high specific stiffness, low density and weight but with very good energy absorbing and damping qualities.

The characteristics are derived from volume images contrasting different material phases obtained by synchrotron micro-tomography. The use of synchrotron radiation for imaging allows one to work with high spatial resolution, a low noiselevel and employing different contrast modes besides the absorption contrast (which is sensitive to the local density and atomic number of the constituents) like holotomography (sensitive to the local electron density), fluorescence tomography (showing the chemical species distribution inside the sample), refraction enhanced tomography (reveals inner surfaces and interfaces) or topo-tomography (displays local crystalline lattice quality). Ideal conditions to apply subsequently a 3d image analysis.

Our analysis is based on two approaches: one via the measurement of the cross-correlation function while the second estimates the probability density function of the distribution of spatial distances between the constituents. The cross-correlation function can be measured using the fast Fourier transform, while the probability density function of the distances is estimated via the Euclidean distance transform.

#### Reference:

A. Rack, L. Helfen, T. Baumbach, S. Kirste, J. Banhart, K. Schladitz, J. Ohser, "Analysis of spatial cross-correlations in multi-constituent volume data", Journal of Microscopy vol. 232, issue 2, 282-292 (2008)

# Image analysis as a tool for the assessment of the weathering of building stones: preliminary results

by Emmanuel Le Trong

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Quantifying and understanding the alteration of building stones under the influence of climatic conditions (weathering) of historical monuments, with the aim to eventually control it is a major issue for the conservation of our cultural heritage. In order to identify the mechanical and chemical processes that occur at the pore scale in the stone, and result in its macroscale decay, we propose an investigation of the modifications of the microstructure of the porous medium. X-ray microtomography provides detailed images of sane and weathered samples which are analyzed using image analysis tools, essentially emanating from the field of mathematical morphology. The stone under study is a particular multi-mineral limestone originating from the Loire valley, the tuffeau, which is mainly composed of silica, calcite and void. The non-trivial step of segmentation of the images (separation of the different phases) will be briefly presented. The structural evolution (in terms of topology and geometry) of the different phases, before and after weathering is then assessed using various classical tools (median axis, granulometry, chords length distributions, etc.). In order to treat full microtomographic images all these tools have been implemented to run on a distributed-memory cluster of computers.

# How to use the watershed transformation for the segmentation of granular materials and porous media obtained by X-ray tomography/SEM

by Vincent Tariel

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The aim of this talk is to propose a simple, generic and robust method to segment images from experimental two- or threedimensionnal images of materials obtained by X-ray tomography and SEM. "Simple" means that this method can be used by one a not specialist of image processing. "Generic" means that this method can be applied in a wide range of materials. Robust means that the extraction is weakly sensitive to pertubation of the segmented parameters. The methodology is:

- 1) If there is a sufficient contrast between the different phases, a classical threshold procedure followed by a succession of morphological filters can be applied,
- 2) If not, and if the boundary needs to be localized precisely, a watershed transformation controlled by seeds is applied. The basement of this transformation is the seeds localization. A seed is associated to a phase such as its localization must respect two properties: inclusion, the seed is included in the phase, and hitting, the seed hits each connected component of the phase.

There are two approaches:

- If it is possible to localize a seed for each phase, a one step watershed is applied to extract in the same transformation all the phases.
- If it is impossible to find a right seed locazition for only one phase, a step by step watershed is applied to extract at each step a phase (the last step is skipped since the last phase is the complementary of the sum of the other phases).

Whatever the algorithm, there are some numerical artefacts coming from the segmentation:

- in tomography, some solids completely surrounded by the liquid phase (levitation),
- in SEM, some holes in the grain phase due to the polishing.

In order to correct these artefacts, an algorithm of hole filling has been apllied. For granular materials, the aim is to individualize each grain with a label. In order to do that, a basic idea is to consider each connected component on the segmented image as a grain. However, an algorithm, based on this idea, would give bad result since there are many grains close one to each other that are connected on the segmented image. To correct this artefact, a split procedure is applied to separate the connected grains. This segmentation has been tested for various complex porous media and granular materials, and allowed to predict various properties (diffusion, electrical conductivity, deformation field) in a good agreement with experimental data.

# Breast density and mammogram aspect as risk factors for breast cancer

by Françoise Clavel-Chapelon

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Most breast cancers originate in the glandular ducts, and the amount of glandular tissue can be estimated with mammography, with dense tissue appearing light on the mammogram. High breast density is one of the strongest known risk factor for breast cancer (Boyd, JNCI, 1995). Women with over 75% of their breasts composed of dense tissues have an estimated 2-6 fold increase in risk, as compared to those with very low breast density (Boyd Lancet Oncol 2005, McCormack CEBP 2006). High mammographic density is common in young women. Density decreases with age and the decline is larger around menopause (Boyd, CEBP 2002). Beyond age, the degree of mammographic density may also depend on several factors such as the number of children, and the age at first birth, body weight, body shape, physical activity, or the hormonal milieu. However, evidence on these questions is not convincing (Oza and Boyd 93, Boyd 1995). Several studies have concluded to a statistically significant increase in mammographic density among women using menopause hormone therapies (MHT) (Greendale, McTiernan, Chiarelli, Christodoulakos, Lundstrom, Rutter, Kilicdag, McNicholas, Ozdemir, Persson, Freedman). In these studies, two categories of hormone therapies were considered, namely estrogens alone or combined with synthetic progestins.

Debate exists about the interaction, on breast cancer risk, between breast density and hormone levels, and between breast density and various other characteristics (age, body mass index, body shape (android/gynoid), physical activity, reproductive factors and endogenous hormone levels). Moreover no data exist to date on the relationship between breast density and use of various French MHT and on the time interval between the initiation of a certain type of MHT and its consequence on breast density. Also, among unresolved questions to date is whether measures other than breast density (i.e. mammographic texture features) are better predictors of the risk. Although some of these issues have been considered previously, most studies have not been large enough or included a long enough follow-up to resolve these concerns. The French E3N cohort study, set up on 100,000 women, followed since 1990, offers the opportunity to clarify these questions, in nested case-controls studies. The review of the literature and the outlines of the E3N project will be presented.

# Using a model of x-ray image formation to quantify breast tissue characteristics

by Chris Tromans

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Since the late 1960s the correlation between radiological features of the tissue structures within the breast and the likelihood of the breast containing, or subsequently developing cancer has been studied in the field of research termed breast density. In terms of cancer risk prediction, many studies have been published showing statistically significant correlations between certain radiographic tissue characteristics observed within a mammographic image, and the subject ultimately developing breast cancer. The techniques exhibiting the strongest correlations are generally those that depend heavily on subjective assessment by skilled readers. Our aim over the past six years has been to develop a fully automated computer algorithm to quantify risk via the characteristics of the tissues present. The two principals issues encountered have been that of the variation in tissue appearance within a mammogram due to the dependence on the x-ray energy characteristics of the radiation used to acquire the image (which is optimised to minimise dose to each patient rather than maintain consistent appearance), and the unclear biological basis of the exact tissue properties which are indicative of risk. Discussion will primarily focus on the approaches we have adopted to overcome these issues: the development of a full software model of the x-ray image formation process, including scattered radiation, and an associated mammography specific measure of radiodensity (analogous to the Hounsfield unit used universally in CT) to provide a normalised measure describing solely the properties of the tissue, independently of the x-ray characteristics used to acquire the image; and the biological studies linking stromal alterations, in particular collagen content, to mammographic appearance and cancer development. Finally our future work which will utilise supervised learning methods to extract the mammographic features indicative of disease and risk from a large clinical database will be presented.

# Mammographic risk assessment: linear structures and texture aspects

by Reyer Zwiggelaar

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The talk will discuss how anatomical linear structures can be extracted from mammographic images and how these might be related to mammographic risk assessment. Secondly, texture segmentation techniques have been used to extract tissue specific regions which again are linked to mammographic risk assessment. The talk will cover a range of mammographic modalities: classical xray mammography, tomosynthesis, and potentially mammographic CT.

# Image-Based Biomarkers of Breast Cancer Risk: Analysis of Breast Density from Digital Breast Tomosynthesis Images

by Predrag R. Bakic

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Identification of women with an increased risk of breast cancer is of high importance, as those women may benefit from modified screening protocols and risk reduction therapy. The current clinical standards, Gail and Claus statistical models, are used to predict the absolute risk of breast cancer over a defined age interval based on standard risk factors, related to personal hormonal and family cancer history. These models perform well on a population level, but are limited in prediction of individual cancer incidence, as the standard risk factors are practically non-modifiable and cannot reflect changes in risk over time.

The breast cancer risk and specific genetic alterations have been linked to phenotypic appearance of breast images, namely breast density and parenchymal texture. Breast density is an independent risk factor of cancer; it is also indicative of changes in modifiable risk factors. In mammography, breast density is quantified as percent density (PD), the percentage of mammogram area occupied by non-fatty, dense tissue. Such defined PD represents a global measure of breast density. Texture analysis of breast parenchyma offers an additional local characterization of breast density spatial distribution.

We have been developing methods for estimating breast density from clinical breast images using various modalities, including mammography and digital breast tomosynthesis (DBT). DBT is a 3D x-ray imaging modality in which high resolution tomographic breast images are reconstructed from multiple low-dose projections acquired within a limited angle range. DBT has potential to replace projection mammography for early breast cancer screening; early DBT clinical trials suggest an improved sensitivity and specificity compared to mammography.

This talk will focus on our current results in estimating breast density from clinical DBT images. The proposed methods for 2D and 3D analysis of dense tissue regions will be discussed, as well as the issues related to DBT acquisition and reconstruction artifacts, and the proposed correction approaches.

# Estimation of the Hurst index on finite frequency bands. Applications in health

by Pierre R. Bertrand

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We deal with Gaussian processes  $\{X(t), t \in R\}$  having zero mean and stationary increments. These processes can also be written using the following harmonizable representations, see [1]:

$$X(t) = \int_{\mathbb{R}} \left( e^{it\xi} - 1 \right) f^{1/2}(\xi) dW(\xi), \quad \text{for all } t \in R,$$
 (1)

where W(dx) is a complex Brownian measure, with adapted real and imaginary part such that the Wiener integral is real valued and

• f is a positive even function, called the spectral density of X, such that

$$\int_{R} \left( 1 \wedge |\xi|^{2} \right) f(\xi) \, d\xi < \infty. \tag{2}$$

When X is a fBm the spectral density follows a power law  $f(\xi) = \sigma^2 |\xi|^{\beta}$  with  $\beta = 2H + 1$  where H denote the Hurst index or in log-log scale  $\ln f(\xi) = \beta \ln |\xi| + \ln \sigma^2$  for all the frequencies from zero to the infinity.

However, for certain applications in medicine or health (heart rhythm or human posture for instance) the behavior of the spectral density differs according to scales and this brings a relevant biological information, see below the example of heart rhythm:

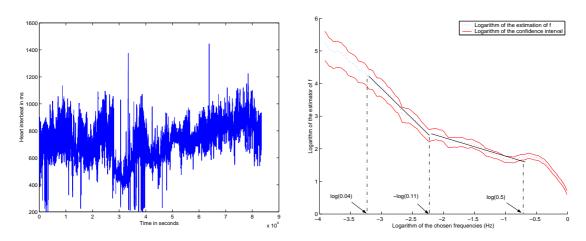


Figure 1: RR interval for a healthy subject during a period of 24 hours (left) and spectral density estimation during the working hours (right).

Our aim is to propose a statistical study of the estimation of the spectral density f on one or several finished frequency bands  $(\omega_k, \omega_{k+1})$  with for eg. k = 1 or 2. We limit ourselves to the case of processes, but we think that this approach could be adapted in superior dimension.

Two cases of observation are considered: first, an idealistic one, where a continuous time path of the process is known, second, a more realistic one, of observation of the process at random times.

By using wavelet analysis, one derives a non parametrical estimator of the spectral density function  $\xi \mapsto f(\xi)$ . One gives Central Limit Theorems (CLT) and estimations of the Mean Integrated Square Error (MISE) in both cases of observation.

Next, one presents numerical experiments and applications to heartbeat time series: one estimates the spectral density on the frequencies bands  $(0.04\,Hz,\,0.15\,Hz)$  and  $(0.15\,Hz,\,0.5\,Hz)$  corresponding respectively to the orthosympathic and the parasympathic nervous systems following [2]. These result are based on real data for healthy subject during a period of 24 hours or on marathon runners during 3 hours.

- [1] Cramér, H. and Leadbetter, M. R. (1967). Stationary and related stochastic processes. Sample function properties and their applications. Wiley and Sons.
- [2] Task force of the European Soc. Cardiology and the North American Society of Pacing and Electrophysiology (1996), Heart rate variability. Standards of measurement, physiological interpretation, and clinical use. *Circulation* 93 (5), 1043-1065.

### Testing fractal connectivity in multivariate long memory processes

by Patrice Abry  $^1$ , H. Wendt  $^1$ , A. Scherrer  $^1$  and S. Achard  $^2$ 

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Within the framework of long memory multivariate processes, fractal connectivity is a particular model, in which the low frequencies (coarse scales) of the interspectrum of each pair of process components are determined by the autospectra of the components. The underlying intuition is that long memories in each components are likely to arise from a same and single mechanism. The present contribution aims at defining and characterizing a statistical procedure for testing actual fractal connectivity amongst data. The test is based on Fisher's Z transform and Pearson correlation coefficient, and anchored in a wavelet framework. Its performance are analyzed theoretically and validated on synthetic data. Its usefulness is illustrated on the analysis of Internet traffic Packet and Byte count time series.

# Anisotropic Random Fields and Their Fractal Dimensions

by Yimin Xiao

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In this talk, we discuss the properties of anisotropy, operator-self-similarity and generalized scale-invariance of random fields and determine the fractal dimensions of various random sets associated with anisotropic (Gaussian or stable) random fields.

# Large intersection properties and Poisson coverings

by Arnaud Durand

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Let  $(X_n, R_n)_{n\geq 1}$  denote a Poisson point process in  $\mathbb{R}^d \times (0,1)$  with intensity  $\mathcal{L}^d \otimes \pi$ , where  $\mathcal{L}^d$  denotes the Lebesgue measure and  $\pi$  is a locally finite measure defined on (0,1). We study the size properties of the random set

$$F = \left\{ x \in \mathbb{R}^d \mid ||x - X_n|| < R_n \text{ for infinitely many } n \ge 1 \right\}$$

in terms of the Hausdorff g-measures for general gauge functions g. We also show that this set is a set with large intersection. This enables us to provide a precise description of the size and large intersection properties of the set of times at which the Hölder exponent of a Lévy process is bounded by a given value.

# Some questions related to boundary value problems in some ramified domains with a fractal boundary

by Yves Achdou

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We consider several aspects related to the study of some boundary value problems in self-similar ramified domains with a fractal boundary, with the Laplace and Helmholtz equations. In particular, for Neumann conditions on the fractal boundary, we propose transparent boundary conditions which allow for the computation of the solutions in subdomains limited by a prefractal curve, i.e. obtained by stopping the geometric construction after a finite number of steps. These conditions involve a nonlocal Dirichlet to Neumann operator which can be obtained by taking advantage of the geometrical self-similarity. The transparent boundary conditions are combined with a multiscale strategy for nonhomogenous Neumann conditions. A self-similar finite element method is proposed and tested.

Theorems of trace on the fractal boundary will be discussed as well: we study the traces of functions belonging to Sobolev space on the ramified domain.

This research has been carried out with N. Tchou (IRMAR). Related published articles are listed below:

- [1] Y. Achdou, C. Sabot, and N. Tchou. A multiscale numerical method for Poisson problems in some ramified domains with a fractal boundary. *Multiscale Model. Simul.*, 5(3):828–860 (electronic), 2006.
- [2] Y. Achdou, C. Sabot, and N. Tchou. Transparent boundary conditions for Helmholtz equation in some ramified domains with a fractal boundary. *Journal of Computational Physics* 220, pp 712-739, (2007).
- [3] Y. Achdou, N. Tchou. Neumann conditions on fractal boundaries Asymptotic Analysis 53, 1-2, pp 61–82 (2007)
- [4] Y. Achdou, N. Tchou. Trace theorems on Domains with Self-Similar Fractal Boundaries Journal de Mathématiques Pures et Appliquées 89, 596-623,2008

### Scaling Properties of the Spread Harmonic Measures

by Denis S. Grebenkov

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Diffusive transport across a semi-permeable interface is ubiquitous in physics, biology, chemistry and industry [1,2]. Diffusing particles first arrive onto the boundary and then explore some neighboring area before reacting or being transferred across this interface. The overall functioning of the interface is then governed by a "competition" between its accessibility (how easy to reach the boundary) and permeability (how easy to cross the boundary). This competition is controlled by two transport parameters: diffusion coefficient D and surface permeability or reactivity W. Their ratio D/W, which is homogeneous to a length, parameterizes a transition from a purely reactive boundary ( $W = \infty$ ) and a purely reflecting boundary (W = 0).

In mathematical terms, the arrival points for diffusion are known to be characterized by harmonic measure. By analogy, one can introduce a family of the spread harmonic measures (parameterized by the ratio D/W) in order to characterize the transfer or reaction points [3-5]. These measures can either be generated by partially reflected Brownian motion, or obtained as solutions of the related boundary value problems for Laplace operator [4-5].

A numerical analysis of the spread harmonic measures on prefractal quadratic von Koch curves has revealed many interesting properties, some of them having remained poorly understood in a rigorous mathematical sense. For instance, the family of the spread harmonic measures exhibits a transition between the harmonic measure (D/W=0) and the Hausdorff measure  $(D/W\to\infty)$ . To our knowledge, this is the first numerical observation of such a transition between two measures having so different multifractal behaviors on fractal sets. Scaling properties of the spread harmonic measures on prefractal boundaries are found to be characterized by a set of multifractal exponent functions depending on the only scaling parameter. A conjectural extension of the spread harmonic measures to fractal boundaries is proposed.

- [1] B. Sapoval, "General Formulation of Laplacian Transfer Across Irregular Surfaces", Phys. Rev. Lett. **73**, 3314 (1994).
- [2] B. Sapoval, "Transport Across Irregular Interfaces Fractal Electrodes, Membranes and Catalysts", in "Fractals and Disordered Systems", Eds. A. Bunde, S. Havlin (Spring-Verlag, Heidelberg, Berlin, New York, 1996).
- [3] M. Filoche, B. Sapoval, "Can One Hear the Shape of an Electrode. II. Theoretical Study of the Laplacian Transfer", Eur. Phys. J. B **9**, 755 (1999).
- [4] D. S. Grebenkov, "Scaling Properties of the Spread Harmonic Measures," Fractals 14, 231 (2006).
- [5] D. S. Grebenkov, "Partially Reflected Brownian Motion: A Stochastic Approach to Transport Phenomena", in "Focus on Probability Theory", Ed. L. R. Velle, pp. 135-169 (Nova Science Publishers, 2006) [Electronic version is available at http://arxiv.org/abs/math/0610080].

# P1. Characterization and Modelling of Microstructures Using Volume Images

by C. Redenbach, O. Wirjadi, H. Altendorf, M. Godehardt, K. Schladitz

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Methods from integral geometry and mathematical morphology yield efficient algorithms for the geometric characterization of microstructures using 3d images. In particular, the intrinsic volumes and their densities are used to describe objects like pores or cells and components, respectively.

Spatial information is particularly important for highly porous structures like foams which are used in an increasing number of application areas, e.g. as heat exchangers, sound absorbers, filters or sandwich panels for lightweight construction. Characteristics like porosity, strut length density or mean strut diameter can be deduced directly from the estimated densities of the intrinsic volumes. Model-based analysis yields quantities such as mean cell volume or mean cell surface area. Empirical distributions of cell characteristics are obtained by an image analytic cell reconstruction and subsequent estimation of the intrinsic volumes of the cells. Based on the estimated values, random tessellation models can be fitted to the observed structures.

As second application example, fibre-reinforced composites are considered. The intrinsic volumes yield fibre volume fraction and fibre length density. The other crucial characteristic is the fibre orientation distribution. Based on anisotropic Gaussian filtering, the orientation tensor, mean fibre directions and anisotropy measures can be computed. As models for fibre systems of non-woven, reinforced polymers and sintered metal fibres, we propose Poisson cylinder processes, hardcore cylinder models and Boolean cylinder shell segment models, respectively.

Simulations of physical properties (e.g. elasticity, heat transfer, sound absorption or filtration) in both the original samples and realizations of models with altered parameters then allow to study relations between the microstructure and the macroscopic behaviour of the materials. This allows for 'virtual' design of foams and fibre systems in composite functional materials.

# P2. Dual Pore Network Simulations Based on High Resolution $\mu$ -CT Images to Calculate the Electrical Properties of Carbonates

by D. Bauer, S. Youssef, E. Rosenberg, S. Bekri, O. Vizika

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Electrical properties of porous media strongly depend on the pore geometry as well as on the topology of the entire pore system. Especially, in carbonates, presenting a bimodal pore size distribution, the topology of the interconnected macropores and the spatial distribution of the micropores and the solid phase play a crucial role in the correct determination of the electrical properties.

The aim of the present work is to calculate the electrical properties of bimodal carbonates by means of a dual pore network approach based on the actual pore geometry and topology obtained from high resolution  $\mu$ -CT. It consists in two major steps: 1) Three phase segmentation of the  $\mu$ -CT images 2) Dual pore network simulations to determine the macroscopic electrical properties.

- Three phase segmentation of the  $\mu$ -CT images: The  $\mu$ -CT images are partitioned in three phases: the macroporosity defined by a pore diameter larger than the  $\mu$ -CT resolution (which is equal to 3  $\mu$ m), the microporous phase characterized by pore diameters smaller than the  $\mu$ -CT resolution, and the solid phase. To achieve this crucial step, the acquisition parameters are adjusted to optimize the image contrast. The originality of this work lies then in the application of a specific filter enhancing the contrast in order to obtain a trimodal grey level histogram. The phase separation was then performed by a simple thresholding operation. The volumic contribution of each phase was deduced. Finally, the equivalent pore network was constructed based on the extracted macroporosity.
- Dual network simulations: The dual network approach combines the classical (single porosity) pore network modelling with the macroscopic properties of the microporosity, supposing that the two systems are in parallel. The electrical properties of the entire porous media can then be determined. The description of the microporosity is based on  $\mu$ -CT images, experimental and numerical data. In order to study the influence of the spatial distribution of the microporosity and the solid phase on the electrical properties two different approaches were applied:
  - Effective medium approximation (microporosity and solid phase are considered as an equivalent homogeneous medium)
  - The microporosity is distributed around a certain percentage of the pore segments of the macroporosity network keeping the entire volume of microporosity constant.

For heterogeneous carbonates, the results of the second approach are in better accordance with experimental data than those obtained by an effective medium approximation. By varying the percentage of segments surrounded by the microporous phase we showed that in double porosity systems the electrical behaviour strongly depends on the spatial distribution and connection of the microporosity.

### P3. Surface Patch Interactions with Topological Guarantees

by M. Couprie, J. Chaussard

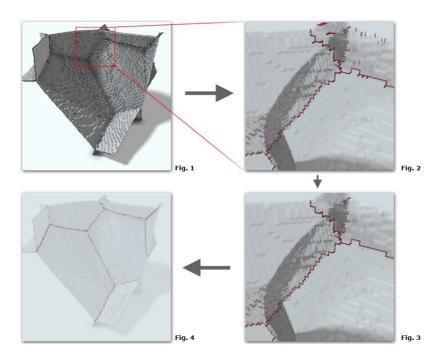
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In applications such as the characterization of the structure of metallic foams from 3D tomograms, a crucial step is the localization of wall intersections. The initial data is a 3D image obtained from tomographic reconstruction. Usually, the segmentation of such images is easy and allows us to identify walls as a set of voxels. Some skeletonisation algorithms can be used to thin these walls, but detection of the voxels located at the wall inter-sections remains a problem. In order to cope with this difficulty, we need a framework where surface inter-sections would be a set of curves (1D elements), and where curve intersections would be a set of points (0D elements). Cubical complexes offer such a framework.

Intuitively, a cubical complex may be thought as a set of elements, or faces, having various dimensions: 3-faces (cubes), 2-faces (squares), 1-faces (edges), and 0-faces (vertices). These faces are glued together according to certain rules [BC08b] (eg, an edge contains two vertices, and a square contains four edges). The collapse operation is an elementary topology preserving transformation which can be seen as a discrete analogue of thinning. It consists in removing two faces (f,g) from a complex X under the condition that f is the only face containing g.

Our method is outlined as follows: let X be the set of voxels of the walls (Fig. 1), we collapse all elements of X while preserving the 2-, 1- and 0-faces on the image borders. The 2-D complex obtained is called Y. Let Z be the set of all edges of Y which are contained in more than 2 squares: the complex Z contains the wall inter-sections, but also contains edges corresponding to local wrinkles which we would like to eliminate (Fig. 2). To do so, we collapse Z until stability is reached, and eliminate isolated vertices (Fig. 3): the resulting complex, only composed of edges and vertices, corresponds to the wall intersections and has the required properties (Fig. 4).



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### P4. Extracting 3D Polyhedral Building Models from Aerial Images

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We describe a model driven approach for extracting simple 3D polyhedral building models from aerial images. The novelty of the approach lies in the use of featureless and direct optimization based on image rawbrightness. The 3D polyhedral model is estimated by minimizing a global dissimilarity function that is based on the bright-ness of the images. This minimization is carried out using the Differential Evolution algorithm - a stochastic and genetic optimizer.

The proposed approach gives more accurate 3D reconstruction than feature-based approaches since it does not involve intermediate noisy data (e.g., the 3D points of a noisy Digital Elevation Maps). We provide experiments and evaluations of performance. Experimental results shows the feasibility and robustness of the proposed approach.

Keywords: 3D polyhedral models, image-based 3D modelling, stochastic and genetic optimizer, direct and featureless approach.

# P5. Hybrid Skeleton Graph Analysis of Disordered Porous Media. Application to Trabecular Bone

by R. Jennane<sup>1</sup>, G. Aufort<sup>1</sup>, C. L. Benhamou<sup>2</sup>

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In this study [1], we present a new method for modelling disordered porous media. Recent work [2] has shown that the line skeleton is a powerful tool for 3D structure characterization. However, as it generates only 1D curves, the geometry of the material is sometimes excessively approximated. Our algorithm is an improved solution as it considers the local shape of each element that composes the structure of the medium. It consists in an efficient combination of curve and surface thinning techniques. Features extracted from the new skeleton contain significant topological and morphological information. A clinical study carried out on trabecular bone samples demonstrates the ability of our method to discriminate between 2 different populations.

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# P6. Spherical Multifractal Texture Synthesis: Computer Graphics, Astrophysics and Analysis

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Algorithms of texture synthesis are more and more efficient (computing time, realism...), specially in computer graphics where textures are synthesized on surfaces using or not a mapping step. We are particularly interested in multifractal textures (scale invariant). We have studied the extension of a 2D model of such textures on a surface with constant curve: the sphere.

This texture synthesis is done directly on the sphere. It then avoids the mapping step, seam or pole artifacts and is totally independent of the spherical grid used. This model has many degree of freedom (pattern, distribution...) and the use of a low-pass filtering preserving the scale invariance allows the synthesis of various textures.

The main application of this model is the validation of a stereographic image processing algorithm using images from the STEREO mission of ESA/NASA. To evaluate the seeming radius of a sphere, the method needs a particular spherical texture. The proposed spheres have then the same statistics as the Quiet Sun (without its violent phenomenon) in the extreme Ultra-Violet. The first results are encouraging and shows the algorithm ability to evaluate known elevation maps.

Finally, having multifractal spherical textures with prescribed characteristics, we have extended a texture analysis to the sphere. Based on a method of multifractal analysis in 2D, we have replaced plane wavelets with spherical wavelets. This analysis captures properly the main behaviour of scale invariance but presents a systematic bias.

We soon expect to extend this algorithm of spherical multifractal texture synthesis to any surfaces.

# P7. A Multiscale Modelling of Fluid Transport within Bones: Consequences of Electrokinetics in the Mechanotransduction of Bones Remodelling Signals

by T. Lemaire, J. Kaiser, V. Sansalone, S. Naïli

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Osteo-articular tissues such as cortical bone are complex saturated porous media that are electrically charged. They are composed of a solid matrix, cells and a fluid phase. The movement of the fluid phase through the pores is referred to as interstitial fluid flow. Although much is suspected about the role of fluid shear stress on biological activity such as growth, adaptation and repair mechanisms [1], relatively little is known about flow characteristics under *in vivo* conditions in the vicinity of the cells. The behaviour of osteoarticular media is governed by different driving forces (hydraulic, chemical, electrical and mechanical).

In this study, a multiphysical description of fluid transport through cortical media is presented. Adapted from [2], a multiscale modelling is used to derive the macroscopic response of the tissue from microscopical information. Bone mechano-sensitive cells are located in the lacuno-canalicular porosity. At this scale, the micro-model is given by the electrohydrodynamics equations governing the electrolyte movement coupled with local electrostatics (Gauss-Poisson equation) and ionic transport equations. Using a change of variables and an asymptotic expansion method (periodic homogenization), a macroscopic description is derived. It contains: (i) the Poisson-Boltzmann problem describing the double-layer potential at the pore scale; (ii) a modified Darcy law involving hydraulic, osmotic and electro-osmotic driving forces; (iii) fluid mass conservation; (iv) charge conservation; (v) ionic species conservation.

This model permits us to propose innovative arguments concerning the role of pericellular matrix located in the pores [3]. In particular, the special role of the microelements forming the pericellular matrix is shown. This matrix not only reduces the interstital flow but also increases electro-chemical couplings effects. In particular, in the vicinity of the osteocytes, the shear stress generated by electrokinetics can not be neglected.

Keywords: Biomechanics, Cortical Bone, Multiphysical Modelling, Multiscale Analysis, Bone Remodelling, Mechanotransduction.

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### P8. Stochastic Volatility and Multifractional Brownian Motion

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We introduce the Multifractional stochastic volatility models. These new parametric models are natural genera-lizations of the Fractional stochastic volatility models studied by Gloter and Hoffmann in [2, 3] since they are of the form  $dZ_t = \Phi(\theta, X_t)dW_t$  where W is a Brownian motion independent on the multifractional Brownian motion (mBm for short) X (see [1, 4]) and  $\theta$  the unknown real-valued parameter. Their main advantage is that they allow to account the variations with respect to time of the local Hölder regularity.

Our main goal is to extend some results in [2, 3] to this new multifrational setting. Namely, under some weak assumptions, we construct an estimator of integrated functional of the volatility starting from the observation of the high-frequency data  $Z(j/n), j=0,\ldots,n$ . We also construct, in the linear case (i.e.  $\Phi(\theta,x)=\theta x$ ), starting from the same data, an estimator  $\hat{\theta}_n^2$  of  $\theta^2$  and give an evaluation of its rate of convergence. It is worth noticing that, when the functional parameter  $H(\cdot)$  of mBm is with values in (1/2,1), then the rate of convergence of  $\hat{\theta}_n^2$  is at least  $n^{-1/(4\{\max_{s\in[0,1]}H(s)\}+2)}$ . The question whether or not  $n^{-1/(4\{\max_{s\in[0,1]}H(s)\}+2)}$  is the optimal rate remains open.

Basically, we use technics which are more or less similar to those in [2, 3], however new difficulties appear in our multifractional new setting. These new difficulties are essentially due to the fact that the dependence structure of multifractional Brownian motion is much more complicated than that of fractional Brownian motion.

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# P9. Mathematical Modelization for Computing the Permeability in Human Cortical Bone

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Many studies (experiments or modelizations) are devoted to human cortical bone. Because of its high complexity, mathematical modelizations are often used. SiNuPrOs [1] is such a modelization which has as main objective to characterize the mechanical behaviour from architectural data and physical characteristics of the basic components which are collagen and hydroxyapatite (Hap). The current developments aim to better characterize the fluid aspects in this multi scale architecture. It is natural to consider bony tissue as a porous medium and this point of view involves the determination of the porosity and the permeability. If porosity does not present a major problem, at least for the order of magnitude, there is a difficulty with the permeability. According to experiments, values vary between 10-13 and 10-23m<sup>2</sup>: it is obvious that the same entities have not been measured.

This paper presents a computation of the permeability tensor based on a concept of multi scale medium corresponding to the scales already introduced in the SiNuPrOs modelization (macroscopic, osteonal and lamellar). In accordance with this modelization, mathematical developments based on the homogenization theory are applied at each of these levels and a numerical scheme is implemented in Matlab to get the values of permeability tensor coefficients. Since many architectural configurations are possible, a reference configuration is associated to the SiNuPrOs modelization. A first study is devoted to the permeabilities at each scale and one notes different order of magnitude (in m²) for each scale: 10-12 for the macroscopic scale, 10-15 for the osteonal one and 10-20 for the lamellar one. A second study concerns the effects, at the macroscopic level, of variations of all parameters related to the fluid modelization: diameters of the Havers and Volkmann channels, diameter of the osteon and distances between two Havers or Volkmann channels. An analysis shows that the most important parameters for the variations of the K33 coefficient are successively the diameter of the haversian channel and the diameter of the osteon. For the K11 (or K22) coefficient, the important parameter is the diameter of the Volkmann channel.

An investigation has been pursued to validate this modelization. A few experimental measurements can be found in literature, for instance [2], and it is possible to point out a configuration for which the theoretical value of the permeability is  $1.2\ 10\text{-}13$ , value which is to be brought closer to the experimental value  $1.1\ 10\text{-}13\ m^2$ .

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### P10. Self-Similar Fractal Mosaics

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A model and a construction algorithm for random mosaics including a fractal component are presented. For technical reasons we assume, a certain self-similarity, which enables us to calculate the Hausdorff dimension of the construction. Also the so-called exact dimension function can be calculated explicitly for our model. Several examples illustrate the results.

#### P11. On a Simulation of Gaussian Random Fields

by S. Mase

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Computer simulations of Gaussian random fields are important in various fields. One of interesting applications is the conditional simulation technique of Geostatistics initiated by G. Matheron. Several methods are proposed to perform large scale Gaussian random fields, but they are approximate in nature or cannot perform large scale simulations. We propose a successive simulation procedure based on Cholesky decompositions of blocked covariance matrices. This method simulates the Gaussian random vector  $X_n$  successively conditional on realizations of preceding vectors  $X_1, X_2, \ldots, X_{n-1}$ . The required memory storage is O(n) in matrix block size unit. The procedure is exact if covariance functions are of finite range. If covariance functions are of infinite range, it is approximate if truncating their ranges appropriately.

### P12. Fractional Stochastic Processes and Fields in Geology

by Alexandre Brouste

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Mountains topographies, faults or fractures surfaces and surfaces that develop by stressenhanced dissolution in crustal rocks present rough and complex morphologies. Statistical analysis of these morphologies could explain thermodynamical mecanism and energy needed for their formation.

The geometrical characterization of rough profiles or surfaces is a widespread problem in various geological examples such as erosion patterns, multiphase fluid percolation in porous rocks, fractures or stylolites. Experimental researches and works on the field are still done, and this problem also ask some mathematical questions: 1. what is the proper model to modelize each surface, 2. how to estimate, with the real data, the parameter of the model and 3. how to simulate such kind of rough surfaces.

# P13. A Pore Scale Modelling of the Dissolution of the Calcite in a Reactive Flow of Water Saturated by $CO_2$ Flowing through a Carbonated Sample

by D. Bernard<sup>1</sup>, M. Giton<sup>1</sup>, P. Benezeth<sup>2</sup>

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# **Practical Informations**

#### Conferences and Coffee breaks

Each talk will take place in the Lavoisier area on the 3rd floor of the "Saints-Pères building":

- Plenary lectures in Lavoisier A
- Invited sessions in Lavoisier A and B
- Poster session in Lavoisier hall.

The coffee breaks will be served daily in the same area in the room Lavoisier C:

- at 10:35
- at 14:00
- at 16:00.

For lunch, we suggest to try one of the various "typical from Paris" restaurants or coffee shops located around the building.

### Computer rooms and Internet access

During the workshop, a computer room and a working room are available for participants' use.

They are located on the 5th floor in the corridor Cordier, labelled as Cordier B and Cordier E.

They are free access and they will be open daily from 8:00 to 19:00.

Moreover, a Wifi connection is available in all the building.

You will receive a password and a login which will give you web access at beginning of the Workshop. To keep in mind:

Internet and	Wifi parameters:
login: .	
passwo	rd:

For further information during the workshop contact Yann Demichel yann.demichel@mi.parisdescartes.fr

### Welcome cocktail

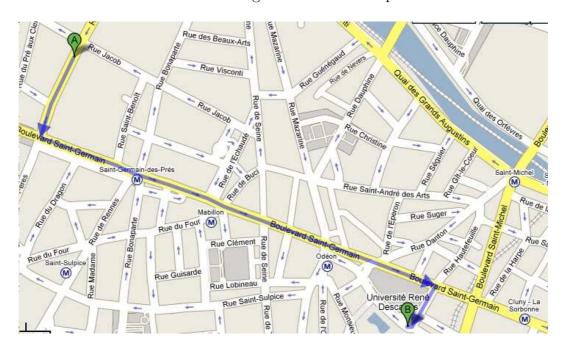
All the participants are kindly invited to a wine and cheese tasting on

#### Monday, January 12 at 19:00

in the "Salle Saint-Germain" at University Paris Descartes head building, located at

12, rue de l'Ecole de Médecine (metro station: Odéon on lines 4 or 10)

If you are willing a 15mn walk, you are invited to meet the organizing committee at 18:30 in the main Hall of the Conference building for a collective departure.



# "Au Revoir" Cocktail

All the participants are invited to a lunch in the Lavoisier C room on

Friday, January 16 at 12:00

### Conference dinner

The Conference dinner will take place on the cruising restaurant "Capitaine Fracasse" which will sail along the river Seine on

#### Wednesday, January 14

Boarding will start at 19:45 and departure will be at 20:30 sharp. Please be sure to be on time!

Access: Swan Island (l'ile aux Cygnes), in the middle of Bir-Hakeim bridge (close to Eiffel Tower), in proximity to Metro Bir-Hakeim (line 6).



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